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About New Nonlinear Properties of the Problem of Nonlinear Thermal Conductivity

Mersaid Mirsiddikovich Aripov
National University of Uzbekistan
Corresponding author email: mirsaidaripov@mail.ru

Maftuha Zafrullaevna Sayfullaeva
National University of Uzbekistan
Email: maftuha87@mail.ru

Abstract---*In this paper we consider a problem of nonlinear heat conduction with double nonlinearity under the action of strong absorption. For which an exact analytical solution is found, analysis of which makes it possible to reveal several characteristic features of thermal processes in nonlinear media. The following nonlinear effects are established: an inertial effect of a finite velocity of propagation of thermal disturbances, spatial heat localization, and finite time effect i.e. existence of a thermal structure in a medium with strong absorption. The following nonlinear effects are observed in the problem under consideration: the inertial effect of an ultimate speed of propagation of thermal perturbations, the effect of spatial localization of heat, and the effect of ultimate time of the thermal structure in an absorption medium.*
Keywords---*degenerate nonlinear, estimate, exact solution, localization, parabolic equation.*

Introduction

In the investigation of the processes of energy transfer in high-temperature environments, a number of their special properties should be taken into account (Zel'Dovich & Raizer, 2002; Bernis & Friedman, 1990). For example, the dependence of heat capacity and the coefficient of thermal conductivity on temperature, it is necessary to take into account the contribution of volume radiation to the energy balance, the processes of exo and endothermic ionization, the leakage of chemical reactions, combustion, etc. The consideration of these factors determines the nonlinearity of the equation of energy transfer (Li & Cui, 2003; Abbasov, 2019). Along with this, one can also take into account convective heat transfer and its influence on the evolution of the process under investigation. The intensive development of the theory of nonlinear transfer was stimulated by studies in plasma physics (Jungel, 2017; Aripov & Sayfullayeva, 2020). Here, fundamental results have recently been obtained, and several nonlinear effects have been discovered, which determine the properties of inertia and localization of thermal.

Problem formulation

Let us consider the following problem about the effect of an instantaneous concentrated source of heat in an incompressible nonlinear medium with a coefficient with double nonlinearity of thermal conductivity of temperature and its grabient in the presence of volume absorption of thermal energy in it, which power depends on the temperature and explicitly on the time according to the power-law (Zmitrenko & Kurdyumov, 1992; Kurdyumov, 1990). Such a non-stationary process of heat conduction is described by the following Cauchy problem for a degenerate quasilinear parabolic equation is not divergence form

$$\frac{\partial u}{\partial t} = u^n \nabla(u^{m-1} \left|\nabla u^k\right|^{p-2} \nabla u) - b(t)u^q,\; u(0,x) = Q_0\delta(x),\; (t>0, x\in R^N) \quad \textbf{(1)}$$

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Here, *u(x, t)* — temperature ,*m,k,p*— the parameter of nonlinearity of the medium: b>0, $bt^{\alpha}u^{q}$ - is the power of volumetric heat absorption; $v(\mathrm{t})$ - a speed of a convective transfer; Q_0 -the value that determines the energy of the heat source at the initial moment; $\delta(x)$— Dirac's delta function that is characterizing the initial temperature distribution of a concentrated heat source placed at the beginning of the coordinate (Martinson & Pavlov, 1972; Mersaid, 2013). To investigating different qualitative properties of the solutions of the problem Cauchy and boundary value problem for a particular value of numerical parameters devoted many works[1-10]. For instance in the case $m=k,\quad v(\mathrm{t})=0,\quad 0<q<1$ by analyzing an exact solution K.Martinson [JVMMF1984] when

$$q=\frac{p-[\mathrm{m}(p-1)-1]}{p-1},\quad 1<m<2,\quad p>m(\mathrm{p}-1)-1$$

establish the following properties of solutions: an inertial effect of a finite velocity of propagation of thermal disturbances, spatial heat localization, and finite time localization solution effect. Let us show that with

$$q=\frac{p-[k(p-2)+n+m]}{p-1},\quad 1<m<2 \quad \textbf{(2)}$$

problem (1) has a definite one. To show it, we consider the class of radially symmetric solutions of the equation, obtained by following

$$u(t,x)=w(t,|\xi|=r),\ \ \xi=\int_0^t v(ydy)-x,\ \ |\xi|=(\sum\nolimits_1^N(\int_0^t v(ydy)-x_i)^{1/2},\ \ x\in R^N \quad \textbf{(3)}$$

Then the unknown function *w (t, r)* satisfies the equation

$$\frac{\partial w}{\partial t}=w^n r^{1-N}\frac{\partial}{\partial r}(r^{N-1}w^{m-1}\left|\frac{\partial w^k}{\partial r}\right|^{p-2}\frac{\partial w}{\partial r})-b(t)w^q,\ w(0,|x|)=u_0(x), \quad \textbf{(4)}$$

Further assuming

$$w(t,r)=a(t)(f(t)-r^{\gamma})_+^{\gamma_1},\ \gamma=p/(p-1),\ \ \gamma_1=(p-1)/(k(p-2)+m+n-2) \quad \textbf{(5)}$$

where, *a (t)*, *f (t)*- are the functions to be defined, and through $(n)_+$, the expression of $(n)_+=\max(0,n)$ is designated (Bhagavannarayana et al., 2011; Bhat & Dharmaprakash, 2002). Calculating the derivatives of the function of *w (t, r)*, we have

$$\frac{\partial w}{\partial t}=\frac{da}{dt}(f(t)-r^{\gamma})^{\gamma_1}-\gamma_1\frac{df}{dt}(f(t)-r^{\gamma})^{\gamma_1-1},$$

$$(r^{N-1}w^{m-1}\left|\frac{\partial w^k}{\partial r}\right|^{p-2}\frac{\partial w}{\partial r})=-(\gamma k\gamma_1)^{p-2}\gamma\gamma_1 a^{(p-2)\mathrm{k}+m}r^N(f(t)-r^{\gamma})^{(k\gamma_1-1)(p-2)+(m-1)\gamma_1+\gamma_1-1}=$$

$$=-(\gamma k\gamma_1)^{p-2}a^{k(p-2)+m}r^N \qquad w(t,r)\in C(Q)$$

If $(\mathrm{k}\gamma_1-1)(p-2)+(m-1)\gamma_1+\gamma_1-1=\gamma_1$

$$(\mathrm{k}(p-2)+m+n)\gamma_1-(p-1)=0$$

Then

$$w^n r^{1-N}\frac{\partial}{\partial r}(r^{N-1}w^{m-1}\left|\frac{\partial w^k}{\partial r}\right|^{p-2}\frac{\partial w}{\partial r}) = -(k\gamma\gamma_1)^{p-2}Na^{k(p-2)+m+n}(f(t)-r^{\gamma})^{(\gamma_1-1)\mathrm{k}(p-2)+(\mathrm{n}+m-1)\gamma_1} - \quad \textbf{(6)}$$
$$-[(\gamma\gamma_1)^{p-1}a^{k(p-2)+n+m}][\gamma(\gamma_1-1)(p-1)+(m-1)\gamma_1]r^{\gamma}[f(t)-r^{\gamma})]^{(\gamma_1-1)\mathrm{k}(p-1)+(m-1)\gamma_1-1}$$

or because

$$(\gamma_1-1)\,\mathrm{k}(p-1)+(m-1)=\gamma_1 \quad \textbf{(7)}$$

expression (6) will be rewritten as

$$w^n r^{1-N}\frac{\partial}{\partial r}(r^{N-1}w^{m-1}\left|\frac{\partial w^k}{\partial r}\right|^{p-2}\frac{\partial w}{\partial r}) = -(\gamma\gamma_1)^{p-1}Na^{k(p-2)+n+m}(f(t)-r^{\gamma})^{\gamma_1} +$$
$$[(\gamma\gamma_1)^{p}a^{k(p-2)+n+m}]r^{\gamma}[f(t)-r^{\gamma})]^{\gamma_1-1} \quad \textbf{(}8\textbf{)}$$
$$w^n r^{1-N}\frac{\partial}{\partial r}(r^{N-1}w^{m-1}\left|\frac{\partial w^k}{\partial r}\right|^{p-2}\frac{\partial w}{\partial r}) - b(t)w^q = -(\gamma\gamma_1)^{p-1}Na^{k(p-2)+n+m}(f(t)-r^{\gamma})^{\gamma_1} +$$
$$+[(\gamma\gamma_1)^{p}a^{k(p-2)+n+m}r^{\gamma} - b(t)a^q][f(t)-r^{\gamma})]^{\gamma_1-1}$$

If $\gamma_1 q = \gamma_1 - 1$

Then, through substituting the calculated expressions into equation (4) we get the following:

$$\frac{da}{dt}(f(t)-r^{\gamma})^{\gamma_1} - \gamma\gamma_1 a(t)\frac{df}{dt}(f(t)-r^{\gamma})^{\gamma_1-1} =$$
$$-(\gamma\gamma_1)^{p-1}Na^{k(p-2)+n+m}(f(t)-r^{\gamma})^{\gamma_1} + [(\gamma\gamma_1)^{p}a^{k(p-2)+n+m}r^{\gamma} - b(t)a^q][f(t)-r^{\gamma})]^{\gamma_1-1}$$

From here we have

$$[\frac{da}{dt}+(\gamma\gamma_1)^{p-1}Na^{\mathrm{k}(p-2)+n+m}](f(t)-r^{\gamma})^{\gamma_1} + [-\gamma\gamma_1 a(t)\frac{df}{dt} - [(\gamma\gamma_1)^{p}a^{k(p-2)+n+m}]r^{\gamma} + \quad \textbf{(9)}$$
$$+b(t)a^q][f(t)-r^{\gamma})]^{\gamma_1-1} = 0$$

Now from here, to define the functions *a(t)*, *f(t)*, we obtain a system of nonlinear differential equations

$$-\gamma\gamma_1 a(t)\frac{df}{dt} + b(t)a^q = (\gamma\gamma_1)^p a^{k(p-2)+n+m} f(t)$$
$$\frac{da}{dt} + (\gamma\gamma_1)^{p-1}[(\gamma\gamma_1 + N)]a^{k(p-2)+n+m} = 0$$
$$\frac{da}{dt} + (\gamma\gamma_1)^{p-1}[(\gamma\gamma_1 + N)]a^{k(p-2)+n+m} = 0,\quad \gamma\gamma_1 = \frac{p}{k(\mathrm{p}-2)+\mathrm{n}+\mathrm{m}} \quad \textbf{(10)}$$

$$\gamma\gamma_1 a(t)\frac{df}{dt} - (\gamma\gamma_1)^p a^{k(p-2)+n+m} f(t) = b(t)a^q \quad \textbf{(11)}$$

And the equation (9) has the following general solution

$$a(t)=[c+(\mathrm{k}(p-2)+n+m)(\gamma\gamma_1)^{p-1}[(\gamma\gamma_1+N)t]^{-\frac{1}{k(p-2)+n+m}}=$$
$$=[c+(\frac{p}{\mathrm{k}(p-2)+n+m})^{p-1}(p+(\mathrm{k}(p-2)+n+m)Nt]^{-\frac{1}{k(p-2)+n+m}}$$

where *c* constant integration.
Rewrite equation (11) as

$$\frac{df}{dt}-b_1(t)\ f=b_2(t) \qquad \mathbf{(12)}$$

Then, taking into account (10) from (11), we have

$$b_1(t)=[c+(\frac{p}{\mathrm{k}(p-2)+n+m})^{p-1}(p+(\mathrm{k}(p-2)+n+m)Nt]^{-1},$$
$$b_2(t)=-\frac{\mathrm{k}(p-2)+n+m}{p}b(t)[a(t)]^{q-1}$$
$$b_1(t)=[(\frac{p}{\mathrm{k}(p-2)+n+m})^{p-1}(p+(\mathrm{k}(p-2)+n+m)N]t^{-1}$$

Hence the solution tending to ∞ at $t\to 0$ has the form of

$$a(t)=[(\frac{p}{\mathrm{k}(p-2)+n+m})^{p-1}(p+(\mathrm{k}(p-2)+n+m)N]t^{-1/(\mathrm{k}(p-2)+n+m)}, \quad (13)$$

The equation (12) is a first-order linear equation. It is integrated. Its overall solution is:

$$f(t)=[c+(\frac{p}{\mathrm{k}(p-2)+n+m-1})^{p-1}(p+(\mathrm{k}(p-2)+n+m)Nt]^{(\frac{p}{\mathrm{k}(p-2)+n+m})^{p-1}(p+(\mathrm{k}(p-2)+n+m)N}[f_0+\int_0^t b_2(y)e^{\int b_1(y)dy}dy]$$

When c=0, we have

$$f(t)=t^{(\frac{p}{\mathrm{k}(p-2)+n+m})^{p-1}(p+(\mathrm{k}(p-2)+n+m)N)}[f_0+\int_0^t b_2(y)e^{\int b_1(y)dy}dy]$$
$$(\sum_1^N(\int_0^t v(y)dy-x_i)^{1/2}=[f(t)]^{(p-1)/p}$$
$$\int_0^t v(y)dy<\infty,\ f(t)<\infty,\ \forall t>0$$

The theorem 1.Let in equation (1)

$$q=\frac{p-[k(p-2)+n+m]}{p-1},\ u_0(x)\le z(0,x),\ x\in R^N$$

where

$$z(t,r)=a(t)(f(t)-r^{\gamma})_+{}^{\gamma_1},\ \gamma=p/(p-1),\ \gamma_1=(p-1)/(\mathrm{k}(p-2)+n+m),$$

and *a(t), f(t)*-are the functions defined above.Then for the problem (1), the phenomenon KSRV takes place.

The theorem 2**.** Let into an Equation(1)

$$q = \frac{p-[k(p-2)+n+m]}{p-1},\ u_0(x) \le z(0,r),\ r \in R, \quad f(t) < \infty,\ t > 0$$

where

$$z(t,r) = a(t)(f(t) - r^{\gamma})_+^{\gamma_1},\ \gamma = p/(p-1),\ \gamma_1 = (p-1)/(\mathrm{k}(p-2)+n+m),$$

and *a(t), f(t)*- are the functions defined above.
Then for the problem (1), the spatial localization of the solution takes place.
Fast diffusion case:

$$k(p-2)+m+n<0$$

The theorem 3. Let in the equation (1)

$$q = \frac{p-[k(p-2)+n+m]}{p-1},\ u_0(x) \le z(0,x),\ x \in R^N$$

where

$$z(t,r) = a(t)(f(t) + r^{\gamma})_+^{\gamma_1},\ \gamma = p/(p-1),\ \gamma_1 = (p-1)/(\mathrm{k}(p-2)+n+m),$$

and *a(t), f(t)* are the functions defined above.
Then for the solution of problem (1), there is a place for the estimate.

$$u(t,x) \le z(t,r),\ r \in R, \quad t > 0$$

The final time of the thermal impulse is due to the influence of the volume absorption of thermal energy is considered medium (Shkir & Abbas, 2014; Isaenko et al., 2005). Indeed, if we consider even the initial temperature distribution of the form $u(x,0) = 0$, then due to the volumetric absorption of heat, the temperature of the medium will decrease over time (Dehghan, 2001; Yang et al., 2008). The following nonlinear effects are observed in the problem under consideration: the inertial effect of an ultimate speed of propagation of thermal perturbations, the effect of spatial localization of heat, and the effect of ultimate time of the thermal structure in an absorption medium (Wei et al., 2006; Mercaldo et al., 2011).

References

Abbasov, I. B. (2019). A Research and Modeling of Wave Processes at the Scattering of Nonlinear Acoustic Waves on Cylindrical Bodies. *International Research Journal of Engineering, IT and Scientific Research*, *5*(5), 32-44.

Aripov, M., & Sayfullayeva, M. (2020). On the new nonlinear properties of the nonlinear heat conductivity problem in nondivergence form. *Bulletin of National University of Uzbekistan: Mathematics and Natural Sciences*, *3*(2), 200-208.

Bernis, F., & Friedman, A. (1990). Higher order nonlinear degenerate parabolic equations. *Journal of differential equations*, *83*(1), 179-206. https://doi.org/10.1016/0022-0396(90)90074-Y

Bhagavannarayana, G., Riscob, B., & Shakir, M. (2011). Growth and characterization of l-leucine l-leucinium picrate single crystal: a new nonlinear optical material. *Materials Chemistry and Physics*, *126*(1-2), 20-23. https://doi.org/10.1016/j.matchemphys.2010.12.040

Bhat, M. N., & Dharmaprakash, S. M. (2002). New nonlinear optical material: glycine sodium nitrate. *Journal of crystal growth*, *235*(1-4), 511-516. https://doi.org/10.1016/S0022-0248(01)01799-7

Dehghan, M. (2001). An inverse problem of finding a source parameter in a semilinear parabolic equation. *Applied Mathematical Modelling*, *25*(9), 743-754. https://doi.org/10.1016/S0307-904X(01)00010-5

Isaenko, L., Vasilyeva, I., Merkulov, A., Yelisseyev, A., & Lobanov, S. (2005). Growth of new nonlinear crystals LiMX2 (M= Al, In, Ga; X= S, Se, Te) for the mid-IR optics. *Journal of crystal growth*, *275*(1-2), 217-223. https://doi.org/10.1016/j.jcrysgro.2004.10.089

Jüngel, A. (2017). Cross-diffusion systems with entropy structure. *arXiv preprint arXiv:1710.01623*.

Kurdyumov, S. P. (1990). Evolution and self-organization laws in complex systems. *Advances in Theoretical Physics*, *134*.

Li, C. L., & Cui, M. G. (2003). The exact solution for solving a class nonlinear operator equations in the reproducing kernel space. *Applied Mathematics and Computation*, *143*(2-3), 393-399. https://doi.org/10.1016/S0096-3003(02)00370-3

Martinson, L. K., & Pavlov, K. B. (1972). The problem of the three-dimensional localization of thermal perturbations in the theory of non-linear heat conduction. *USSR Computational Mathematics and Mathematical Physics*, *12*(4), 261-268.

Mercaldo, A., Peral, I., & Primo, A. (2011). Results for degenerate nonlinear elliptic equations involving a Hardy potential. *Journal of Differential Equations*, *251*(11), 3114-3142. https://doi.org/10.1016/j.jde.2011.07.024

Mersaid, A. (2013). To properties of solutions to reaction-di ff usion equation with double nonlinearity with distributed parameters. *Journal of the Siberian Federal University. Mathematics and Physics* , *6* (2).

Shkir, M., & Abbas, H. (2014). Physico chemical properties of l-asparagine l-tartaric acid single crystals: A new nonlinear optical material. *Spectrochimica Acta Part A: Molecular and Biomolecular Spectroscopy*, *118*, 172-176. https://doi.org/10.1016/j.saa.2013.08.086

Wei, Z., Li, G., & Qi, L. (2006). New nonlinear conjugate gradient formulas for large-scale unconstrained optimization problems. *Applied Mathematics and computation*, *179*(2), 407-430. https://doi.org/10.1016/j.amc.2005.11.150

Yang, L., Yu, J. N., & Deng, Z. C. (2008). An inverse problem of identifying the coefficient of parabolic equation. *Applied Mathematical Modelling*, *32*(10), 1984-1995. https://doi.org/10.1016/j.apm.2007.06.025

Zel'Dovich, Y. B., & Raizer, Y. P. (2002). *Physics of shock waves and high-temperature hydrodynamic phenomena*. Courier Corporation.

Zmitrenko, N. V., & Kurdyumov, S. P. (1992). Time Reversion of Processes in Dissipative Systems. *Modern Physics Letters B*, *6*(01), 49-54.