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Measurement of Small Deformations with a Mach-Zehnder Interferometer

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Abstract---*We show a method for measuring small deformations of solids using the principle of the Max-Zender interferometer. In the interference pattern, we find an analytical expression for the dependence of the maximum displacement of the displacement, i.e., the change in the order of interference, on the deformation force of the solid. We will show you how to determine the Joung modulus for a solid body using specific expressions. The proposed method shows that the measurement accuracy of the deformation of solids is about 10^{-5} m. We describe the analytical expressions obtained for the interference order in two- and three-dimensional graphical form.*
Keywords---*coherent wave, deformation, Joung modulus, Mach - Zehnder interferometer, path difference*

Introduction

The scheme of the Mach - Zehnder interferometer is a device for solving problems requiring splitting of waves, mainly electromagnetic, into parts and their further reduction together. Interferometers are often used to observe interference - the increase and decrease in the amplitude of two or more waves when they are superimposed on each other (Khashan & MA, 1983). Interference is easiest to observe when waves with the same properties interact, therefore, to create them, they use one emitter, the waves of which are separated and then again reduced (Diddams & Diels, 1996). The main components of a Mach - Zehnder interferometer are a radiation source, two opaque mirrors, and two semitransparent ones (Figure 1). Opaque ones completely reflect radiation, and translucent ones - only part of the light, letting in the rest (Bor et al., 1990).

When the radiation source is turned on, a beam is an incident on the first semitransparent mirror, which is then split into two derivatives. The first of them pass through the mirror, and the second is reflected and moves perpendicular to the original ray. Further, two derived beams are reflected from the opaque mirrors and directed to the second semitransparent mirror. It also splits both beams into two parts. The halves of the derived rays begin to overlap (Beck & Walmsley, 1990). They originate from the same source and therefore have approximately the same frequency and total phase difference, as a result of which, when interacting with each other, they create a series of light and dark stripes in the sensors or on the screen - the result of interference. Dark stripes will appear where the waves of rays have extinguished each other, light ones - where they have intensified (Beck et al., 1991).

The Mach-Zehnder interferometer is a versatile device suitable for many tasks. It is very often used to measure changes in refractive indices and gas flux density. Therefore, the Mach-Zehnder interferometer is applicable for visualizing air flows. It can also serve as a device for refracting light when creating holograms (Sainz et al., 1994). Mach-Zehnder interferometers are extremely valuable for studying one of the paradoxical phenomena in modern physics - quantum entanglement. Entangled qubits when using a Mach-Zehnder interferometer are very well separated from each other, thus avoiding their direct interaction. An unusual application of the Mach-Zehnder interferometer is to visualize air flows (Kovács et al., 1995). In this paper, we explore the possibility of measuring

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very small deformations and determining the Jung modulus of solids using the Mach-Zehnder interferometer (Pedrotti, 2008; Zhao et al., 2019; Hernández et al., 2017).

Main Part

Basic equations of interference

Interference of light - the addition in space of two or more coherent waves, in which at different points there is an increase or decrease in the amplitude of the resulting wave. The equations of coherent waves joining at a point $\boldsymbol{M}$ in space is as follows:

$$x_1 = A_1 \cos\left(t - \frac{s_1}{\upsilon_1}\right) \quad \text{and} \quad x_2 = A_2 \cos\left(t - \frac{s_2}{\upsilon_2}\right) \tag{1}$$

from the cosine law, we can find the resulting vibration amplitudes following as

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos\delta \tag{2}$$

and given that the intensity of light is proportional to the square of its amplitude $I \sim A^2$, (2) equation appears as follows

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\delta \tag{3}$$

The last term in equation (3) is called the interference term because the resulting intensity can increase or decrease depending on this term. Therefore, an interference pattern arises depending on the phase difference δ in this term (Savelev, 1970). And where δ is the phase difference of oscillations excited at point $\boldsymbol{M}$ which is equal to

$$\delta = \omega\left(\frac{s_2}{\upsilon_2} - \frac{s_1}{\upsilon_1}\right) = \omega\left(\frac{s_2}{c/n_2} - \frac{s_1}{c/n_1}\right) = \frac{2\pi\nu}{c}(L_2 - L_1) = \frac{2\pi}{\lambda_0}\Delta \tag{4}$$

Where $\upsilon = c/n$ - is the speed of light in a medium, $\omega = 2\pi\nu$ - is the cyclical frequency of light, and $\lambda_0 = c/\nu$ - is the wavelength of light in a vacuum. The multiplication of the geometric path length $\boldsymbol{s}$ of the light wave in a given medium by the refractive index of this medium $\boldsymbol{n}$ is called the optical path length;

$$L = s \cdot n \tag{5}$$

It can be seen from equation (4) that the resulting phase difference δ is dependent on the values taken by the optical path difference Δ. If Δ the difference in optical paths is equal to an integer number of wavelengths in a vacuum (an even number of half-waves)

$$\Delta = \pm m\lambda_0 = \pm 2m\frac{\lambda_0}{2} \qquad (m = 0, 1, 2....) \tag{6}$$

In this case, the phase difference (4) takes the following values $\delta = \pm 2\pi m$, and the resulting intensity in the expression (3) leads to an increase (Granato et al., 1958; Cooper et al., 2001). Therefore, expression (6) is called the maximum condition for interference. If the optical path difference is equal to an odd number of half-waves

$$\Delta = \pm(2m+1)\frac{\lambda_0}{2} \qquad (m = 0, 1, 2....) \tag{7}$$

In this case, the phase difference (4) takes the following values $\delta = \pm(2m+1)\pi$, and the resulting intensity in the expression (3) leads to a decrease. Therefore, expression (7) is called the minimum condition for interference.

Method for measuring small deformations on a Mach-Zehnder interferometer

In this section, we considered a method for detecting very small deformations of a solid using the Mach-Zehnder interferometer. For this, we need to determine the expression for the path difference that occurs in the interferometer due to the deformation of the solid attached to the interferometer (Galdieri et al., 2020; Wiesauer et al., 2005). As can be seen from figure 1.

$$\frac{\delta}{L/2} = \sin\alpha \quad \text{and} \quad \frac{\Delta_i}{h} = \sin\alpha \qquad (8)$$

From the equations (8), the following expression can be found

$$\Delta_i = \frac{2\delta h}{L} \qquad (9)$$

Where Δ_i - is the difference in optical paths caused by the deformation of only one side of a solid body. However, as can be seen from figure 1, due to the deformation of the solid body, a path difference occurs on both sides of it. Therefore the difference in total optical paths is expressed as follows

$$\Delta = 2\Delta_i = \frac{4\delta h}{L} \qquad (10)$$

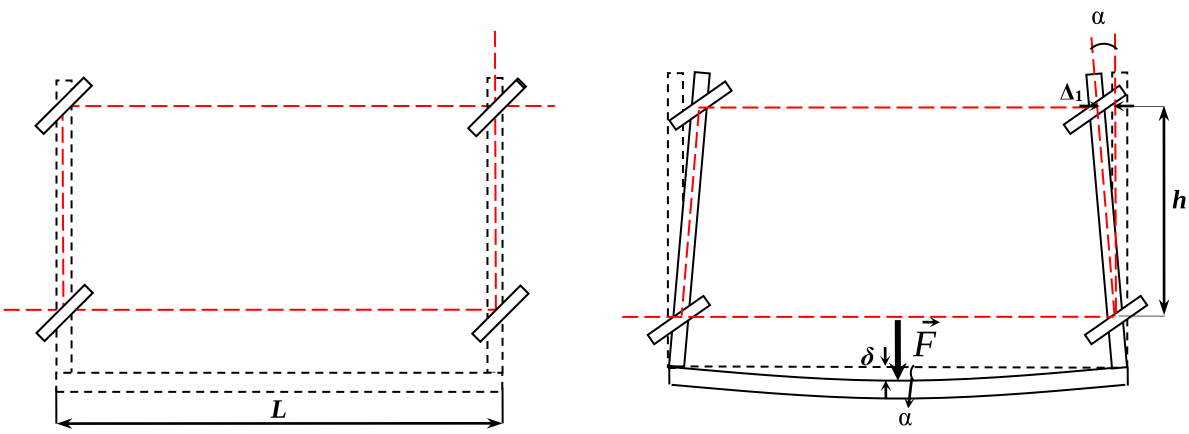

Figure 1. Schematic of the Mach-Zehnder interferometer, which measures small deformations

Where δ - is the magnitude of the deformation. It is defined in the (Sivukhin, 1979), literature as follows

$$\delta = \frac{FL^3}{48EI} \qquad (11)$$

Where E - is the module Jung, I - is the moment of inertia of a solid body, L - is the length of the deformable solid body. The moment of inertia is given in the following literature (Sivukhin, 1979)

$$I = \frac{ab^3}{12} \qquad (12)$$

Using formulas (10), (11), and (12), the following expression can be found

$$\delta = \frac{FL^3}{4Eab^3} \quad \text{and} \quad \Delta = \frac{hFL^2}{ab^3E} \qquad (13)$$

From the maximum condition for interference (6), we can find the relationship between the deformation force and the interference pattern

$$m = \frac{\Delta}{\lambda} = \frac{hFL^2}{ab^3\lambda E} \qquad (14)$$

the change in the order of interference with the change force can be determined from the following equation

$$\Delta m = \frac{hL^2}{ab^3\lambda E}\Delta F \qquad (15)$$

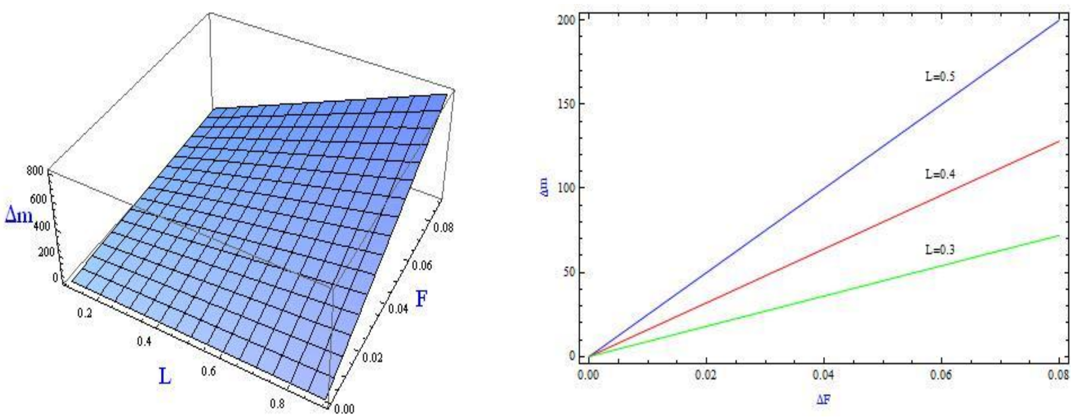

Figure 2. The dependence of the deformation force on the order of interference

Figure 2 shows that the force acting depends on the change in the interference pattern as a result of the deformation of the solid attached to the interferometer (Shin et al., 2000; Chekroun et al., 2009). This method of studying deformations is also distinguished by its sensitivity to deformations in order 10^{-5} m namely

$$\Delta m \sim \Delta F \cdot 10^5 \qquad (16)$$

Using the (15) expression, the Jung modulus for various solids can be determined with great precision, e.g.

$$E = \frac{hL^2}{ab^3\lambda}\frac{\Delta F}{\Delta m} \qquad (17)$$

Results and Discussions

Using the method described above, deformations of the order of 10-5 m can be detected. The formula (15) shows that increasing the distance between the mirrors $\Delta m \sim h \sim L^2$ can further improve the measurement accuracy of the interferometer. The presence of $\Delta m \sim 1/\lambda$ the same expression indicates the high accuracy of experiments with low-wavelength light. Equation (17) shows that the above considerations can also be applied to Young's modulus, which determines the mechanical properties of a solid (Goranson & Adams, 1933; Macías et al., 2018).

Conclusion

In this paper, we have studied the method of measuring very small-order deformations using the Mach-Zehnder interferometer and determining the Jung modulus for a solid body. We also found an analytical expression for the dependence of the deformation force of a solid body on the interference order. Those identified expressions were analyzed using graphs (Weisser et al., 1999; Taraphdar et al., 2010). Analyzes show that even very small deformations in the order of 10^{-5} m can be measured very accurately using the Mach-Zehnder interferometer.

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