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Calculate Central Limit Theorem for the Number of Empty Cells after Allocation of Particles

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Abstract---*This work belongs to the field of limit theorems for separable statistics. In particular, this paper considers the number of empty cells after placing particles in a finite number of cells, where each particle is placed in a polynomial scheme. The statistics under consideration belong to the class of separable statistics, which were previously considered in (Mirakhmedov: 1985), where necessary statements for the layout of particles in a countable number of cells were proved. The same scheme was considered in (Asimov: 1982), in which the conditions for the asymptotic normality of random variables were established. In this paper, the asymptotic normality of the statistics in question is proved and an estimate of the remainder term in the central limit theorem is obtained. In summary, the demand for separable statistics systems is growing day by day to address large-scale databases or to facilitate user access to data management. Because such systems are not only used for data entry and storage, they also describe their structure: file collection supports logical consistency; provides data processing language; restores data after various interruptions; database management systems allow multiple users.*
Keywords---*central limit theorem, particle placement, particle, polynomial scheme, separable statistics*

Introduction

This paper considers a scheme of random placement of particles in cells. The same scheme was considered in Mirakhmedov (1988); Kolchin et al. (1976), in which the conditions of asymptotic normality were established for the number of cells containing r particles. In Asimov (1982), in the arrangement of two types of particles, three-dimensional symmetric separable statistics PC ($\mu_{00}, \sum_{r\geq 0}\mu_{0r}, \sum_{r\geq 0}\mu_{r0}$) was studied, where $\mu_{r_1 r_2}$ - are the number of cells containing r_1 particles of the first and r_2 particles of the second type, respectively. In the present work, we studied the distribution of particles of s types in a finite number of cells (Mirakhmedov, 1987; Mirakhmedov, 1989; Popova, 1968). The considered value is μ_0 - the number of empty cells after placing all *s* types of particles in which each type of particles is placed in a polynomial scheme (Mitchell et al., 2006; Valjarević & Petrović, 2020;

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Indahyati & Sintaasih, 2019; Benedicta, 2021). The asymptotic normality of statistics μ_0 is proved, and an estimate of the remainder term in the central limit theorem is obtained (Ivchenko & Levin, 1978; Mikhailov, 1981).

Main Results

Let s types of particles be allocated independently of each other and consistently in N cells, the number of l -type particles be equal to n_l , each of them falls into the cell with the number m with the probability (Hall, 1984; Dedecker & Rio, 2000).

$$P_{lm} > 0 \quad , \quad m = \overline{1; N} \quad , \quad P_{l1} + ... + P_{lN} = 1 \quad , \quad l = \overline{1; s}$$

Consider the following random variable $\mu_o(s)$ - the number of empty cells after the allocation of all $n_1, \ \dots \ , n_s$ particles. Evidently, each type of particles is allocated by the polynomial scheme, i.e. the random vector.

$$\eta_l = (\eta_{l1}, ..., \eta_{lN})$$

Can be set by the conditional distribution of the random vector $\xi_l = (\xi_{l1} , \ \dots \ , \xi_{lN})$, where we have:

$$Z(\xi_{lm}) = \prod (n_l p_{lm}) \quad , \quad m = \overline{1; N} \quad , \quad l = \overline{1; s} \quad ,$$

For independent random variables ξ_{lm}

$$\text{Set } \ \xi_l^{(s)} = (\xi_{l1} , \ \dots \ , \xi_{lN}) \ , \ \lambda_{lm} = n_l P_{lm} \quad ,$$

$$\alpha_l = {n_l}/{N} \quad , \quad \lambda_m = \lambda_{1m} + \ldots + \lambda_{sm} \quad ,$$

$$A_n(s) = \sum_{m=1}^{N} e^{-\lambda_m} \quad , \quad \gamma_e = \frac{1}{n_l} \sum_{m=1}^{N} \lambda_{lm} e^{-\lambda_m} .$$

$$\sigma_N^2(s) = \sum_{m=1}^{N} e^{-\lambda_m} \left[1 - e^{-\lambda_m} - \sum_{m=1}^{N} \alpha_l \gamma_l^2 \right]$$

We suppose that for each $l = 1, \ \dots, \ s$

$$\max N_{p_{lm}} \le C_{\circ} \quad , \quad \ln \alpha_l \le \varepsilon N$$

Theorem**.** There exists $C(s)$ such that

$$\Delta_N^{(s)}(y) = \left| P\left(\frac{\mu_0(s) - A_n(s)}{\sigma_N(s)} < y \right) - \Phi(y) \right| \le C(s) \left[\sigma_N^{-1}(s) + \sum_{m=1}^{N} (n_m)^{-1} \right]$$

Proof

We consider the random variable:

$$g(\xi_m^{(s)}) = f(\xi_m^{(s)}) - e^{-\lambda_m} + \sum_{l=1}^{s} \gamma_l (\xi_{lm} - \lambda_{lm}),$$

Where $f(\xi_m^{(s)}) = f(\xi_{l1}, \dots, \xi_{ls})$ - a random function of non-conventional integer arguments.
According to the work we have:

$$\Delta_N^{(s)}(y) \le C(s)[\frac{1}{\sigma_N^3(s)} \sum_{m=1}^{N} E\left|g(\xi_m^{(s)})\right|^3 + \sum_{j=1}^{s} \frac{1}{n_j}] \qquad (1)$$

We will cush the value $\sigma_N^2(s)$ in the form:

$$\sigma_N^2(s) = \sum_{m=1}^{N} (1-(1+\lambda_m)e^{-\lambda_m})e^{-\lambda_m} + \sum_{m=1}^{N} \sum_{l=1}^{S} \lambda_{lm} (e^{\lambda_m} - \gamma_l)^2$$

And the random value $g(\xi_m^{(s)})$ we write in the form:

$$g(\xi_m^{(s)}) = f(\xi_m^{(s)}) + e^{-\lambda_m} \sum_{e=1}^{S} \xi_{lm} - (1+\lambda_m)e^{-\lambda_m} + \sum_{e=1}^{S} (\gamma_l - \lambda_m)(\xi_{lm} - \lambda_{lm})$$

Then:

$$\begin{aligned} E\left|g(\xi_m^{(s)})\right|^3 &\le 4E\left| f(\xi_m^{(s)}) + e^{-\lambda_m} \sum_{j=1}^{s} \xi_{jm} - (1+\lambda_m)e^{-\lambda_m} \right|^3 + \\ &+ 4s^2 \sum_{j=1}^{s} \left|\gamma_j - e^{-\lambda_m}\right|^3 E\left|\xi_{jm} - \lambda_{jm}\right|^3 = 4\Delta_{1m} + 4s^2\Delta_{2m} \end{aligned} \qquad (2)$$

The random variable $\xi_{1m} + \dots + \xi_{sm}$ has Pousson distribution with parameter λ_m. Moreover, the distribution of random variable $f(\xi_m^{(s)})$ coincides with distribution of random variable $f(\xi_{1m} + \dots + \xi_{sm})$, where:

$$f(0)=1 \text{ and } f(y)=0, \text{ if } y>0.$$

Thus

$$\begin{aligned} \Delta_{1m} &= E\left| f(\xi_{1m} + \dots + \xi_{sm}) + e^{-\lambda_m}(\xi_{1m} + \dots + \xi_{sm}) - (1+\lambda_m)e^{-\lambda_m} \right|^3 = \\ &= e^{-\lambda_m}\left(1 - e^{-\lambda_m}\left(1+\lambda_m\right)\right)^3 + \lambda_m^4 e^{-4\lambda_m} + \\ &+ \sum_{j=2}^{\infty} \left| j - 1 - \lambda_m \right| e^{-4\lambda_m} \cdot \frac{\lambda_m^j}{j!} = \Delta_{1m}^{'} + \Delta_{1m}^{''} + \Delta_{1m}^{'''} \end{aligned} \qquad (3)$$

Since $(1+u)e^{-u} \le 1$ for $u > 0$, then

$$\sum_{m=1}^{N}\Delta_{1m}^{'} \le \sum_{m=1}^{N} e^{-\lambda_m}(1-e^{-\lambda_m}(1+\lambda_m)) \le \sigma_N^2(s) \qquad (4)$$

From the fact that $u^2e^{-2u} \le 1$ and $\frac{1}{2}u^2e^{-u} \le 1-(1+u)e^{-u}$ we receive:

$$\sum_{m=1}^{N}\Delta_{1m}^{''} \le \sum_{m=1}^{N}\lambda_m^2 e^{-2\lambda_m} \le 2\sum_{m=1}^{N}(1-(1+\lambda_m)e^{-\lambda_m}) \le 2\sigma_N^2(s) \qquad (5)$$

Let $\sum\limits_{\lambda_m \le 1}$, $\sum\limits_{\lambda_m > 1}$ means summation by those m , for which $\lambda_m \le 1$ and $\lambda_m > 1$ respectively. We have:

$$\sum_{\lambda_{m\le 1}}\Delta_{1m}^{'''} = \sum_{\lambda_{m\le 1}} e^{-\lambda_m^3}\left[E\left(\xi_{1m}+...+\xi_{sm}-1-\lambda_m\right)^3-\left(1+\lambda_m\right)^3 e^{-\lambda_m^3}+\lambda_m^4 e^{-\lambda_m}\right]$$

Let ξ Poisson random variable with parameter. Then for any $l \ge 2$

$$E\left(\xi-\lambda\right)^l = \sum{}^{'}\frac{l!\lambda_m^{k_2+...+k_l}}{k_2!...k_l!(2!)^{k_2}...(l!)^{k_l}} \qquad (6)$$

In $\sum^1$ sum over all nonnegative integers k_2 , ... , k_l such that $2k_2+...+lk_l = l$.

So:

$$E\left|\xi-\lambda\right|^l = c\left(l\right)\left(\lambda^{l'/2}+\lambda\right)^{l/l'} \qquad (7)$$

Where:

$$l' = \begin{cases} l \ , & if \quad l = 2k \\ l-1 \ , & if \quad l = 2k+1 \quad , k \in z \end{cases}$$

Hense for $\lambda_m \le 1$ we have: $E\left|\xi_{1m}+...+\xi_{sm}-1-\lambda_m\right|^3 \le c\left(\lambda_m+1\right) \le c$

Therefore, given that when $\lambda_m \le 1$

$\left(1+\lambda_m\right)^3 = 1+3\lambda_m+3\lambda^2+\lambda^3 \le 1+3\lambda_m+C3\lambda_m^2$, then similarly (5) we have:

$$\sum_{\lambda_m \le 1}\Delta_{'m}^{'''} \le c\sum_{m=1}^{N}\lambda_m^2 e^{-2\lambda_m} \le c\sigma_N^2(s) \qquad (8)$$

Further due to (6) and (7) and inequalities between moments we set:

$$\sum_{\lambda_m>1}\Delta'''_{1m}\le\sum_{\lambda_m>1}e^{-3\lambda_m}E\left|\xi_{1m}+\ldots+\xi_{sm}-\lambda_m-1\right|^3\le$$
$$\le\sum_{\lambda_m>1}\left[E\left(\xi_{1m}+\ldots+\xi_{sm}-\lambda_m-1\right)^4\right]^{3/4}\cdot e^{-3\lambda_m}\le \qquad (9)$$
$$\le c\sum_{\lambda_m>1}\lambda_m^{3/2}e^{-3\lambda_m}\le c\sum_{\lambda_m>1}\lambda_m^2e^{-2\lambda_m^2}\le c\sigma_N^2(s)$$

Summing up the ratios (1)-(9) we receive:

$$\sum_{m=1}^{N}\Delta_{1m}\le C\delta_N^2(s) \quad (10)$$

Estimate the value $\sum_{m=1}^{N}\Delta_{2m}$. Since $\lambda_{jm}\le C_0\alpha_j$, that for $\alpha_j\le 1$, receive $\lambda_{jm}\le C_0$.
Given this ration we have:

$$\sum_{m=1}^{N}\left|\gamma_j-e^{-\lambda_m}\right|^3E\left|\xi_{jm}-\lambda_{jm}\right|^3\le C\sum_{\lambda_{jm}\le1}(\gamma_j-e^{-\lambda_m})^3\sum_{l=1}^{\infty}\left|i-\lambda_{jm}\right|^3\pi_i\left(\lambda_{im}\right)+$$
$$+\sum_{\lambda_{jm}>1}(\gamma_j-e^{-\lambda_m})^3\left[E\left|\xi_{jm}-\lambda_{jm}\right|^4\right]^{3/4}\le$$
$$\le C\left[\sum_{\lambda_{jm}\le1}(\gamma_j-e^{-\lambda_m})^2\left(E\left(\xi_{jm}-\lambda_{jm}\right)^3+2\lambda_{jm}{}^3e^{-\lambda_{jm}}\right)+\right. \qquad (11)$$
$$\left.+\sum_{\lambda_{jm}>1}(\gamma_j-e^{-\lambda_m})^2\lambda_m{}^{3/2}+\sum_{\lambda_{jm}>1}\left|\gamma_j-e^{-\lambda_m}\right|^3\lambda_{jm}{}^{3/2}\right]\le$$
$$\le c\sigma_N^2(s)+\sum_{\substack{\lambda_{jm}>1\\ \alpha_j>1}}\left|\gamma_j-e^{-\lambda_m}\right|^3\cdot\lambda_{jm}^{3/2}$$

When obtaining the last inequality, the relations were taken into account (6) and (7).

Let: $\lambda_{jm}>1$, $\lambda_j>1$. Then $\gamma_j<1/\alpha_j$.

Therefore: $$\left|\gamma_j-e^{-\lambda_m}\right|\lambda_m^{1/2}\le\gamma_j\lambda_m^{1/2}+1\le\frac{\sqrt{c_0}}{\sqrt{\alpha_j}}+1\le\sqrt{c_0}+1$$

Hence:

$$\sum_{\lambda_{jm}>1}\left[\left|\gamma_j-e^{-\lambda_m}\right|\lambda_m^{1/2}\right]^3\le(\sqrt{c_0}+1)\sum_{m=1}^{N}\left(\gamma_j-e^{-\lambda_m}\right)^2\lambda_{jm}\le$$
$$\le(\sqrt{c_0}+1)\sigma_N^2(s)$$

From here and from (11) receive:

$$\sum_{m=1}^{N} \Delta_{2m} \le c\sigma_N^2(s) \qquad (12)$$

This from (1), (10), (12) we have:

$$\sum_{m=1}^{N} E\left|g\left(\xi_m^{(s)}\right)\right|^3 \le c(s)\sigma_N^2(s) \qquad (13)$$

From (1) and (13) follows the theorem. Theorem is proved.

Conclusions

In summary, the demand for separable statistics systems is growing day by day to address large-scale databases or to facilitate user access to data management (El-Zonkoly, 2011; Lee et al., 2015; Chaloupka et al., 1974; Ardell, 1972). Because such systems are not only used for data entry and storage, they also describe their structure: file collection supports logical consistency; provides data processing language; restores data after various interruptions; database management systems allow multiple users to work in parallel (Degond et al., 1999; Cesati & Trevisan, 1997). The main requirement for considers the number of empty cells after placing particles in a finite number of cells, where each particle is placed in a polynomial scheme. Systems is to safely store external data and to respond to the request of the user to satisfy it. This requires the completeness of the information stored in the medical database to maintain the integrity of the data (Rakhimov et al., 2021). If the GPU is the Device, then the size of the partition and blocks is limited. Each block is executed on a separate multiprocessor independently of other blocks. Therefore, the width and height of the block are determined by the maximum number of simultaneously processed threads on the multiprocessor. In turn, when executed, the blocks are divided into bundles of 32 threads, which are launched in accordance with the commands of the multiprocessor thread control unit. Communication between threads in a block is done using shared memory and barrier thread synchronization (Rakhimov et al., 2021).

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