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A revisit to some basic concepts for the complete understanding of physical quantities and their mathematical handling

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Abstract---*This paper is concerned with an unambiguous understanding of physical quantities at a basic level where one is to deal with mathematical symbols linked to scientific concepts. To form an unambiguous understanding of physical quantities and their mathematical handling, a revisit has been made to some of the well-known notations/concepts such as: (i) symbols for physical quantities, (ii) quantity calculus, (iii) principle of unit-independence, and (iv) various kinds of physical equations. The fact that prior knowledge about quantity calculus, the principle of unit-independence, and the notations for physical quantities is essential to mathematically handle physical quantities has been emphasized, along with considering the importance of numerical value equations for physical quantities.*
Keywords---*Physical quantity, Fundamental quantity, Derived quantity, Unit of physical quantity, Physical equation.*

Introduction

In a recent paper, Bhattacharjee (2020), the discovery of the existence of a misleading procedure of handling physical quantities in many places of the traditional literature has been disclosed. It has been found that the long-running procedure of solution of formula-based problems with given numerical data in physics, as well as in different branches of science and engineering is ambiguous because such a procedure did not give appropriate honor to the actual identity of a physical quantity, which, in general, has got a magnitude as well as a unit. Again, the discovery of ambiguity in the traditional norms of labeling physical quantities along the axes of coordinates in drawing data-based graphs has been reported in Bhattacharjee (2022), along with offering an unambiguous way of axes labeling in the said context. To avoid any further confusion in such regard as well as to form an unambiguous understanding of physical quantities and their mathematical handling at a basic level where one is to deal with mathematical symbols linked to scientific concepts, a revisit has been made in this paper regarding some of the well-known notations/concepts such as: (i) symbols for physical quantities, (ii) quantity calculus, (iii) principle of unit-independence, and (iv) various kinds of physical equations. The fact that prior knowledge about quantity calculus, the principle of unit-independence, and the notations for physical quantities is essential to mathematically handle physical quantities has been emphasized, along with considering the importance of numerical value equations for physical quantities (Anderson, 2017).

Physical quantities are often found to appear in the mathematical statement of a law in Physics or a physical equation. So, gathering an unambiguous knowledge about physical quantities as well as their mathematical handling is essential to all who are concerned with the study of Physics in particular and science and engineering in general. But unfortunately, in many traditional resources such as Champion & Davy (1952), Halliday et al. (2013), Kimball (1923), Matveev (1989), Verma (2008), much less attention has been paid to the unambiguous understanding of physical quantities. For example, there is no reflection of the symbolic representation of physical quantities as per international convention in those available resources, although the symbols of physical quantities are always involved in the mathematical statement of a law in Physics or a physical equation. It would be worth mentioning here that this fundamental point has been considered in (Mills et al., 1994; Halliday & Resnick, 1962; Gehbauer et al., 1979). The long-used literature Champion & Davy (1952), Halliday et al. (2013), Kimball (1923), Matveev (1989),

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Verma (2008), does not pay any attention to the principle of unit independence of physical quantities (Morikawa & Newbold, 2002), as well as to the quantity calculus (Taylor, 2018) although they play an important role for the unambiguous understanding of physical quantities and their mathematical handling. Furthermore, there exists no discussion in Champion & Davy (1952), Halliday et al. (2013), Kimball (1923), Matveev (1989), Verma (2008), regarding the classification of physical equations (Gehbauer et al., 1979).

On account of the lack of incorporation of any discussion relating to the symbolic representation of physical quantities, the principle of unit-independence of physical quantities, and the quantity calculus, how could one grasp up the exact meaning of the statements “a body of mass m”, and “let F be the force applied to a particle”, etc., which are normally found to exist in many places of the long-used literature? In the statement “a body of mass m”, what is m? Does m stand for the numerical value of mass of the body in a particular system of units? If not, is it the product of a numerical value and a unit? Without having any prior knowledge about the symbolic notations for physical quantities and the principle of unit-independence of physical quantities, it is very difficult to answer the aforesaid queries regarding the statement “a body of mass m”. After having had a prior knowledge about the symbols for physical quantities, quantity calculus and the principle of unit-independence of physical quantities, it can be boldly said that in the statement “a body of mass m”, m stands for the physical quantity mass and hence m is the product of a numerical value and a unit thereby dispensing away all confusions regarding the exact meaning of the statement “a body of mass m”. Similarly, one can easily understand the exact meaning of F in the statement “let F be the force applied to a particle”, and so on.

Symbols for Physical Quantities

It is well-known that, according to international convention, physical quantities are denoted by appropriate symbols. Such symbolic representation of a physical quantity has been used to write down the mathematical statement of a law or a physical equation. It is therefore very much essential to know before the start of derivation of a physical equation or writing down the mathematical form of a law, which symbol stands for what physical quantity. Although the symbolic representation of physical quantities has been considered in Gehbauer et al. (1979), Halliday & Resnick (1962), Mills et al. (1994), such a discussion regarding the notations for physical quantities is missing in many traditional resources (Champion & Davy,1952; Halliday et al., 2013; Kimball, 1923; Matveev, 1989; Verma, 2008). Furthermore, no priority has been given in Champion & Davy (1952), Halliday et al. (2013), Kimball (1923), Matveev (1989), Verma (2008), regarding the incorporation of the statement of the principle of unit-independence (Morikawa & Newbold, 2002). As a result, incorporation of statement like “let m be the mass of a body” or “let us consider a body of mass m” before starting the derivation of a physical equation as is usually found to exist at many places of the prevailing literature appears to be very much confusing in respect of knowing whether the notation “m” here stands for the physical quantity mass until and unless one is well aware of the principle of unit-independence (Morikawa & Newbold, 2002). Interested readers may get them well-equipped about the symbols for physical quantities appearing in different branches of physics from (Mills et al., 1994).

Quantity Calculus

Quantity calculus, Mills et al. (1994); Morikawa & Newbold (2002); Taylor (2018), or in a true sense, the Quantity algebra, is an algebraic system to deal with quantities where the symbol of a quantity represents the product of the numerical value and its unit. Here, the term ‘calculus” should mean in its broader sense “a system of computation,” and it should not therefore be thought of in the ordinary sense of differential calculus and integral calculus of Mathematics.

In this algebraic system, the rules of ordinary algebra are equally applicable to quantities as they are to numbers, and at the same time, they are also equally applicable to units of measurement because the units, by their very definitions, are also quantities in their own right. Thus, in quantity calculus, a physical quantity is expressed in the form:

Physical quantity = Numerical value of the physical quantity × Unit of the physical quantity
or, simply, Physical quantity = Numerical value × Unit

or, $\frac{\text{Physical quantity}}{\text{Unit}} = \text{Numerical value}$

The following simple example gives some idea about the use of quantity calculus.

Density (ρ) of water at 4^0C in SI base units can be expressed as:

$$\rho = 1000 \text{ kg m}^{-3}$$

Again, in CGS base units, the density of water at 4^0C can be expressed as:

$$\rho = 1 \text{ g cm}^{-3}$$

It then follows from above that

$$\frac{\rho}{\text{kg m}^{-3}} = 1000$$

$$\text{and} \quad \frac{\rho}{\text{g cm}^{-3}} = 1$$

Thus, according to quantity calculus, the ratio $\frac{\rho}{\text{g cm}^{-3}}$ has the significance that it represents the number of grams in one cubic centimeter. In a similar manner the ratio $\frac{\rho}{\text{kg m}^{-3}}$ corresponds to the number of kilograms in one cubic meter.

Such an algebra has been internationally recommended for the mathematical handling of physical quantities, but has not yet been found to be implemented in many traditional resources as it could be. It is of fundamental importance in modern science. More information about quantity calculus is available in (Taylor, 2018).

Principle of Unit-Independence

As per the International Convention, each physical quantity is denoted by a standard symbol. Such a symbol of a physical quantity never stands for the measure of the physical quantity in terms of particular units. This statement is known as the principle of unit-independence of physical quantities, or simply the principle of unit-independence (Morikawa & Newbold, 2002). Saying "let m be the mass of a body", "let F be the force acting on a particle" is based on the principle of unit-independence.

Different Kinds of Physical Equations

As per traditional literature Gehbauer et al. (1979), there exist primarily three different kinds of physical equations that appear in the study of science and engineering. These are: (i) Quantity equation, (ii) Numerical value equation, and (iii) Equation between units.

If a physical equation consists of symbols, each of which stands for a physical quantity (which is the product of a numerical value and a unit), the equation will be called a quantity equation. For example, the equation F = ma, where the symbols F, m, and a stand for the physical quantities force, mass, and acceleration, respectively, will be called a quantity equation (Posdziech & Grundmann, 2007).

Again, if a physical equation consists of the numerical value of every physical quantity involved in the equation in a particular system of units, the equation will be called a numerical value equation. For example, the equation: V = RI, where V stands for the numerical value of potential difference in volts, R stands for the numerical value of resistance in ohms, and I stands for the numerical value of electric current in amperes, is a numerical value equation. If a physical equation describes a numerical relation between units, the equation will be called an equation between units. For example, 1cal = 4.2 J is an equation between units.

Importance of the Numerical Value Equation for Physical Quantities

The following points are in favour of the fact that Numerical value equations for physical quantities have considerable importance in the study of science and engineering.

(i) Since the quantity equation involves quantities each of which is a physical quantity having a numerical value along with a relevant unit, such an equation in a particular system of units is never in irredundant or minimal form. On the other hand, since the numerical value equation for physical quantities involves only the numerical values of the physical quantities in a particular system of units, such a numerical value equation for physical quantities is always in the minimal or irredundant form, having compliance with the ordinary algebraic equation. That is why making use of such an equation should be given top priority in the study of science and engineering.

(ii) On account of their simplified nature, the numerical value equations for physical quantities are easier to handle mathematically as compared to the quantity equations.

(iii) The widespread use of the numerical value equation for physical quantities in different branches of science and engineering also indicates the fact that such an equation is of paramount importance for the relevant field of study.

Conclusion

To enhance the teaching learning process regarding understanding of physical quantities, their mathematical handling as well as to enrich the long-running resources Champion & Davy (1952), Halliday et al. (2013), Kimball (1923), Matveev (1989), Verma (2008) in the said context, a revisit has been made in this paper to the standard terminologies: (i) symbols for physical quantities, (ii) quantity calculus, (iii) principle of unit-independence of physical quantities, by making use of the standard literature (Mills et al., 1994; Morikawa & Newbold, 2002; Taylor, 2018). A prior knowledge about symbols for physical quantities as per the international convention, quantity calculus, as well as the principle of unit-independence of physical quantities, is essential for an unambiguous understanding of physical quantities, and their mathematical handling has been clearly explained and emphasized. Finally, the various kinds of physical equations that the long-running literature Champion & Davy (1952), Halliday et al. (2013), Kimball (1923), Matveev (1989), Verma (2008) are missing have been considered.

It is felt that the material presented in this paper must have educational value, and it will serve as a useful material to form the necessary foundation for the unambiguous understanding of physical quantities and their mathematical handling.

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