Problem-Based Learning and Authentic Assessment on Conceptual Understanding and Ability to Solve Mathematical Problems (Among Students of Politeknik Negeri Bali)

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Abstract

This study was aimed at finding out: 1) the difference in concept understanding and ability to solve mathematical problems between the students who were facilitated with problem-based learning model and those facilitated with conventional learning model; 2) the difference in concept understanding and ability to solve mathematical problems between the students who were facilitated with authentic assessment and those who were facilitated with conventional assessment; and 3) the effect of interaction between learning model and assessment model on concept understanding and ability to solve mathematical problems. This study was conducted at Politeknik Negeri Bali by using the 2x2 factorial experimental design. The sample consisted of 110 students. The data was collected using a test which has been validated, proved to have a reliability coefficient of 0.83 for concept comprehension test, and 0.88 for the problem-solving test. Furthermore, the data were analyzed with two-way Manova. The result of analysis showed significantly that: (1) there was a difference in concept understanding on mathematical problem-solving ability between the students facilitated with problem-based learning model and those facilitated with conventional model; (2) there was a difference in concept understanding and problem-solving ability between the students who were facilitated with authentic assessment, and those facilitated with conventional assessment; and (3) there was an effect of interaction between learning model and assessment on conceptual understanding and ability to solve mathematical problems.
1. Introduction

Mathematics, a universal science underlying the development of modern technology, has an important role in various disciplines and develops the human mind power. Rapid development in the field of information and communication technology currently is based on the development of mathematics in the field of number theory, algebra, analysis, discretionary theories of opportunity and mathematics. To master and create technology in the future requires a strong mastery of mathematics from an early age.

Mathematics as one of the subjects is considered to play a very important role, therefore, the purpose of mathematics learning at school and college is to develop students’ mathematics power. The power of mathematics is referred to as the five standards of the mathematical learning process, namely: understanding, reasoning, communication, connection, and solving mathematical problems (NCTM, 2000: 52). Conceptual understanding is the ability to understand concepts, operations, and relations in mathematics (Kilpatrick, Swafford, & Findell, 2001: 118). Problem-solving skills are the students’ ability to use mathematics activities to solve problems in mathematics, problems in other subjects, and problems in their daily lives (Soedjadi, 1994: 36).

The understanding concept gives a basis for a meaningful mathematics learning since the concept of mastery will ease students to learn and solve the mathematics problem (Kilpatrick, Swafford, & Findell, 2001:118). The ability to solve the problem is the general goal of mathematics course, and even as the nuclear of mathematics learning. This ability is a basic skill in math and facilitates students to be literate in the subject (Romberg, 1994: 288).

Understanding the concept and solving the problem of mathematics is a very important aspect in every learning either in school or university level, as both competences give students opportunity to solve problems related to daily lives, work, and other sciences (Devlin, 2010).

Concept understanding and mathematics problem solving have a very important role in education both for students of Polytechnic and society, thus the endeavor of education quality improvement particularly mathematics learning should be given a serious attention both from private or the government of Indonesian.

The Government of Indonesia has made efforts to improve the quality of education in Indonesia. These efforts have been legally and formally mandated through Law Number 20 of 2003 on National Education System. Furthermore, it was operated with Regulation Number 19 of 2005 on National Education Standards, namely standard of content, process, competency of graduates, educators and education personnel, facilities and infrastructure, management, financing, and assessment. These efforts have been realized in every educational unit. However, many research results explain that the understanding and problem-solving ability of math from students in Indonesia is as yet low. Internationally and nationally, mathematics has been considered not to be able to achieve satisfactory results and students’ mathematics achievement is still low (Hadi, 2017: 3). Some indicators were that the average score of the SMA / SMK / MA National Examination (NE) 2015 of Mathematics subject in Science program decreased by 1.23 points (Mendikbud, 2015: →). The result of TIMSS 2011 competition and PISA in 2012, as well as 2015, showed that Indonesian students gained low rank (Balitbang, 2015). Their science and technology mastery was considered slower than those of other country in Southeast Asia, such as Singapore, Malaysia, and Thailand (Hadi, 2017: 3).

The low quality of education in Indonesia as indicated by TIMSS and PISA is certainly caused by many factors. One of them is that Indonesian students are generally poorly trained in solving math problems (Ruseffendi, 2006: 10; Hadi, 2017: 3). This can happen because the problem-solving activities in learning mathematics have not been used as the main activity. Another factor is that the learning model and assessment used by lecturers. Learning model has an important role in creating the success of the learning process. Assessment as one of the main pillars determines learning activities (Sani, 2016: 8).

The low quality of education in Indonesia as shown by TIMSS and PISA is caused by many factors. One of them is caused by the fact that Indonesian students have less practice in solving mathematical problems (nonroutine problems). Another factor is the learning model and assessment used by lecturers. Learning model has an important role in creating the success of the learning process. Assessment as one of the main pillars determines learning activities.

Improvement in students’ understanding and ability to solve problems can be done through an innovative and effective teaching (Kemendikbud, 2014: 51). Innovative learning is an interpretation of the learning process that is comprehensive with regard to various modern learning theories based on learning innovation. The teaching process is done systematically and structurally using an effective model and method which fits the characteristics of the course.

Problem-based Learning (PBL) is a teaching model which makes problems as the basis for the students to learn (Sadia, 2014: 3; Eggen & Kauchak, 2012: 324; Barrows, 1996: 8; Arends, 2007: 276). They point out some characteristics of PBL, namely: 1) the teaching process is student-centered; 2) the teaching process occurs in small groups; 3) the teacher/lecturer plays the role of facilitator or guide; 4) the problem posed is the stimulus for learning;
5) new information is obtained autonomously (self directed learning); and 6) the problem serves as a means to develop problem-solving skill (Sadia, 2014: 73). Thus, indirectly the attainment of the ability to solve the problem can be developed with PBL.

PBL gives many benefits to the students in developing high order thinking abilities such as thinking critically, discovering and using learning resources, working cooperatively, and learning long life. Barrel states that PBL can prepare the students to become inquirers, problem solvers, and critical thinkers and creative thinkers in meeting the complex challenges. PBL helps students develop skills needed for their success at colleges and working world (Barel, 2010).

PBL, when compared to other teaching models, is a constructivist teaching model which is complex which gives a great opportunity for the formation of learning autonomy. PBL is very effective for developing high order thinking skills that are critical thinking and creative thinking. A complex, contextual and ill-structured problem will give an opportunity to the students to develop an analytic, evaluative, and reflective thinking ability and their creativity in probing various pieces of information, in developing a solution and creating various sources to solve the problem that they have to solve (Sadía, 2016: 76).

Fatade, Mogari, & Arigbabu (2013, 27-43) point out that there is a significant difference between a pretest and a posttest in advanced mathematics learning achievement between the students who were taught with PBL and those who were taught with the traditional method, the advanced mathematics learning achievement of the students taught by PBL was significantly better than that of those taught by traditional method. PBL has an effect on mathematics learning and it improves the students’ understanding and their ability to use concepts in the real life, PBL is more effective for teaching mathematics (Padmavathy & Mareesh, 2013: 47-51).

Assessment is an integral part of the teaching process. Assessment is often regarded as one of the three main pillars which are critical in the teaching activities. The three pillars are planning, implementation and assessment. If the three pillars are synergetic and sustainable, then they will determine very much the quality of the teaching. Hence, assessment has to be designed and implemented according to the planning and implementation of the teaching. The assessment system has to be developed in line with the development of teaching model and strategy. The implementation of teaching model in the classroom in an effort to develop the students’ mathematics problem-solving ability will, of course, become increasingly optimal with the presence of an accurate assessment. Wolf states that the quality of a good teaching has to be followed by an implementation of a good assessment too (Badmus, 2007: 10). Assessment is the important part of teaching and a good teaching will not be successful without a good assessment. A good assessment, in general, will proceed with a good teaching and learning process. In general, the teaching and learning process will proceed effectively if it is supported by an effective assessment too (Sani, 2016: 8).

Regarding the problem above, there is a need for an assessment system to lead the whole process and steps of learning that are related to the development of competence. The assessment practice which is needed is the one which is meaningful, involving the students and meeting the function of improving and empowering the students. Thus, an assessment practice that only stresses on an individual performance target has to be avoided.

Mueller (2005, 1-7) points out that authentic assessment is a form of assessment with which students are asked to perform real-world tasks that demonstrate meaningful application of essential knowledge and skills. Sani (2016, 24), that authentic assessment is a type of assessment which orients the students to demonstrate skills and competencies which are needed to solve problems and situations encountered in the real world. The competencies are a combination of skills that are based on knowledge and are implemented with a suitable attitude. A person cannot be said to be competent yet if his or her attitude in demonstrating his or her skill is not fit and proper (Sani, 2016: 26). Authentic assessment is fit for assessing PBL teaching process (Eggen & Kauchak, 2012: 348).

PBL model and authentic assessment are very fit to be used with vocational schools, considering that one of the functions of this educational institution is to prepare the students to encounter the real world, by making the students aware of the expectations desired from them, the challenges that they will meet, and the abilities they need to master (Dryden, 2002: 34). In PBL, the students are given contextual problems. Through the problems, the teacher relates the material taught to the students’ real world. This approach will encourage the students to make a relationship between the knowledge they have mastered and its implementation in the daily life. As the consequence, the teaching will be more alive, the students become motivated to solve the problems they face. Thus, it will be easier for them to understand the concepts contained in the problems. As the result, their mathematics learning achievement will improve.

By combining PBL with authentic assessment, it is expected that the students can solve problems given as the process to master existing mathematical concepts. Through PBL the students are invited to solve contextual problems. The students are encouraged to make relationships between knowledge that they possess and the implementations in the daily life. Their discussion results are then made in the form of a simple report which is presented as one form of

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authentic assessments. The authentic assessment which is combined with PBL is aimed at enhancing activities and motivation of the students in learning and it gives them an opportunity to always assess themselves so that they can find out the level of mastery of the materials that they have reached. With the improvement in activities and motivation in learning and the knowledge about the relationship between mathematics and the real world, it will be easy for the students to understand the existing concepts. As the result, the students’ mathematics learning achievement will increase.

The main problems investigated in this study are: 1) Is there any difference in concept understanding and ability to solve mathematical problems between the students who are facilitated with PBL model and those who are facilitated with conventional teaching model?; 2) Is there any difference in concept understanding and ability to solve mathematical problems between the students who are facilitated with authentic assessment model and those who are facilitated with conventional assessment model?; and 4) Is there any interaction effect between the teaching model with the assessment on conceptual understanding and ability to solve mathematical problems?

2. Research Methods

2.1 Research Subject

This study was conducted at the Mechanical Engineering Department of Politeknik Negeri Bali, using quasi-experimental design by measuring two factors in the 2x2 factorial version. There were 110 students involved as research participant distributed in 4 classes. Random sampling was used to select the sample and groups in the experiment. The experiment design is shown in Figure 1.

![Figure 1. The 2x2 Factorial Experimental Design](image)

Note:

A = Teaching Models
B = Assessment
A1 = Problem-Based Learning Model
A2 = Conventional Teaching Model
B1 = Authentic Assessment
B2 = Conventional Assessment
Y1 = Concept Understanding
Y2 = Problem Solving Ability

2.2 Data Collecting and Instrument

Data were collected using a mathematical concepts understanding test, and mathematical problem-solving skills, developed by the researchers themselves. Each instrument has been tested both theoretically and empirically. Theoretically, content validity is tested using techniques developed by Aiken’s. The Aiken’s index of each test ranged from 0.5 to 0.95. Empirically validity and reliability are tested using a classical test approach. The validity of the concept comprehension test and the problem-solving test is tested using Product Moment correlation, and it is proved that each test item has a coefficient of $r \geq 0.3$. The reliability of each test is calculated using Cronbach Alpha and each has a reliability coefficient of 0.83 and 0.88.

2.3 Data Analysis

Data were analyzed descriptively, qualitatively, and multivariate (Manova) 2x2 factorial analysis. Before analyzing the analysis requirements were tested: multivariate normality test, homogeneity of variant between groups,
homogeneity of covariance-matrix variables, and multicollinearity test. Test results show that these assumptions have been met.

3. Results and Analysis

The mean (M) and standard deviation (SD) of conceptual understanding and ability to solve mathematical problems and the category of each variable after the treatment are presented in Table 1.

Table 1
General description of data of measurement results of conceptual understanding and ability to solve mathematical problems

<table>
<thead>
<tr>
<th>Group</th>
<th>Concept Understanding</th>
<th>Ability to Solve Mathematical Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M and SD</td>
<td>Category</td>
</tr>
<tr>
<td>A₁B₁</td>
<td>82.53 5.33</td>
<td>Good</td>
</tr>
<tr>
<td>A₂B₂</td>
<td>78.43 4.62</td>
<td>Good</td>
</tr>
<tr>
<td>A₁B₂</td>
<td>75.644 4.27</td>
<td>Good</td>
</tr>
<tr>
<td>A₂B₂</td>
<td>69.689 6.66</td>
<td>Fair</td>
</tr>
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</table>

Table 1 shows that after the treatment, the average score of concept comprehension and math problem-solving ability of the student group facilitated by PBL learning model and authentic assessment (A₁B₁) is highest compared to other student groups. The average score of the student group facilitated by conventional learning model and authentic assessment (A₂B₁) was higher than that of the student group facilitated by PBL model and conventional assessment (A₁B₂). The average score of the student group facilitated by conventional learning model and conventional assessment (A₂B₂), the lowest.

Hypotheses tested using Manova. All hypotheses were tested consecutively, the recapitulation of which can be seen in Table 2 below.

Table 2
Results of the 2 x 2 factorial MANOVA for concept understanding and ability to solve mathematical problems data

<table>
<thead>
<tr>
<th>Source</th>
<th>Dependent Variable</th>
<th>Type III Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corrected Model</td>
<td>Concept Understanding</td>
<td>3776.635</td>
<td>3</td>
<td>1258.878</td>
<td>40.775</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Ability to Solve Problems</td>
<td>2457.487</td>
<td>3</td>
<td>819.162</td>
<td>37.551</td>
<td>.000</td>
</tr>
<tr>
<td>Intercept</td>
<td>Concept Understanding</td>
<td>628017.808</td>
<td>1</td>
<td>628017.808</td>
<td>20341.357</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Ability to Solve Problems</td>
<td>607100.708</td>
<td>1</td>
<td>607100.708</td>
<td>27830.221</td>
<td>.000</td>
</tr>
<tr>
<td>A</td>
<td>Concept Understanding</td>
<td>909.336</td>
<td>1</td>
<td>909.336</td>
<td>29.453</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Ability to Solve Problems</td>
<td>869.216</td>
<td>1</td>
<td>869.216</td>
<td>39.846</td>
<td>.000</td>
</tr>
<tr>
<td>B</td>
<td>Concept Understanding</td>
<td>2574.500</td>
<td>1</td>
<td>2574.500</td>
<td>83.387</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Ability to Solve Problems</td>
<td>1246.671</td>
<td>1</td>
<td>1246.671</td>
<td>57.149</td>
<td>.000</td>
</tr>
<tr>
<td>A * B</td>
<td>Concept Understanding</td>
<td>235.990</td>
<td>1</td>
<td>235.990</td>
<td>7.644</td>
<td>.007</td>
</tr>
<tr>
<td></td>
<td>Ability to Solve Problems</td>
<td>303.035</td>
<td>1</td>
<td>303.035</td>
<td>13.891</td>
<td>.000</td>
</tr>
<tr>
<td>Error</td>
<td>Concept Understanding</td>
<td>3272.638</td>
<td>106</td>
<td>30.874</td>
<td>2312.331</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Ability to Solve Problems</td>
<td>21.814</td>
<td>106</td>
<td>21.814</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>Concept Understanding</td>
<td>634832.000</td>
<td>110</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ability to Solve Problems</td>
<td>611578.000</td>
<td>110</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Discussion

Based on the recapitulation shown in Table 3.2, it can be informed that the findings of this study are as follows.

From the source of the effect of teaching model (A) on mathematical conceptual understanding was found F statistical value of 29.453 (p < 0.05) and the effect of Teaching Model (A) on the ability to solve mathematical problems was found F statistical value = 39.846 (p < 0.05). These results prove that significantly, there is a difference in understanding the concept and problem-solving ability of mathematics between students who are taught using the PBL model with conventional learning model. PBL model is better than conventional learning model, both in achieving comprehension of mathematics concept (M_{A1} = 78.45; M_{A2} = 71.92) and ability of problem solving mathematics (M_{A1} = 77.26; M_{A2} = 71.12).

From the source of the effect of Assessment (A) on the mathematical concept, the understanding was found F statistical value of 147.674 (p < 0.05) and the effect of Assessment (B) on the ability to solve mathematical problems was found F statistical value of 83.273 (p < 0.05). These results prove that significantly there are differences in understanding of concepts and mathematical problem-solving abilities of the students who were assessed using authentic assessment models with the conventional assessment. Authentic assessment model is better than conventional assessment model, both in achieving understanding of mathematical concepts (M_{B1} = 80.46; M_{B2} = 69.91) as well as mathematical problem solving ability (M_{B1} = 77.84; M_{B2} = 70.53).

From the source of the effect of the interaction between teaching model and assessment model (A*B) on the mathematical concept, the understanding was found F statistical value of 9.423 (p < 0.05) and the effect of A*B on the ability to solve mathematical problems was found F statistical value of 19.034 (p < 0.05). These results prove that significantly there is an interaction effect between the learning model and the assessment model on the understanding of concepts and problem-solving skills of mathematics.

According to Savery & Duffy (1995, 31-38), pedagogically that PBL with its constructivist framework has the characteristics such as 1) understanding is obtained from the interaction and scenario of problem and learning environment; 2) struggle with problem and inquiry process creates cognitive dissonance that stimulates learning; and 3) knowledge is created through collaboration of social negotiation and evaluation of the presence of a point of view.

In PBL, teaching focuses on the problem selected so that the students do not only learn concepts related to the problem that becomes the center of attention but also have a learning experience that is related to the skill to apply the scientific method in solving problems and develop critical thinking pattern. The pedagogy of PBL helps the students by showing and clarifying the thinking method and the richness of the cognitive structure and process involved in it. In addition, in PBL the students can become “self-directed learners” who are willing to understand and learn, formulate learning need, ability to select and use the best learning resources. Self-directed learners are individuals who direct themselves in the teaching and learning process. Sadia emphasizes that in PBL situation the students get learning experience to develop a high thinking skill, skill for analyzing and solving problems and developing autonomy and competitiveness. In such a situation the students learn how to use an interactive process and assessment of what they know, to collect information and to collaborate in evaluating a hypothesis based on the data that have been collected (Sadia, 2014: 66). A similar idea is expressed by Ibrahim and Nur who say that in PBL situation the students can be engaged in a high order thinking and problem-solving in a situation-oriented toward real-world problems. This includes learning how to learn (Ibrahim dan Nur, 2000: 43). The students are expected to have whole knowledge of a material formulated in the problem, have a positive attitude, and skill in a step-by-step and sustainable way. The mental activities of the students are required to understand concepts, principles, and skills through a situation and the problem presented at the beginning of the lesson. The students understand the concepts and principles of a material from working and learning the situation or problem given through investigation, inquiry, and problem-solving. The students develop the concepts and principles based on their own ability which integrate skill and knowledge that they have understood before (Ibrahim dan Nur, 2000: 43).

While conventional teaching model is a teaching model which is teacher-centered. Coleman states that conventional teaching is an assimilation of information or classroom teaching with the characteristics such as 1) information is obtained through symbolic sources such as teacher or books; 2) assimilation or organization of information in such a way that the students can understand general principles; 3) use of general principles in specific cases; and 4) application of general principles in new conditions (Santyasa, 2014: 1). Santyasa gives an important note for conventional teaching.

<table>
<thead>
<tr>
<th>Corrected Total</th>
<th>Concept Understanding</th>
<th>7049.273</th>
<th>109</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ability to Solve Problems</td>
<td>4769.818</td>
<td>109</td>
</tr>
</tbody>
</table>

Note: A = Teaching Model; B = Assessment
that states that conventional teaching stresses the importance of activities of obtaining knowledge from symbolic information sources, such as listening to teacher/lecturer or reading books. Source of information has a strong effect on learning process (Santyasa, 2014: 2).

In implementing conventional teaching model the teacher/lecturer’s authority tends to dominate the class. The teacher/lecturer has to demonstrate knowledge or skill to be taught or trained to the students step by step. The teacher/lecturer’s role is required that he or she should become an interesting model for the students, material to be developed according to his or her taste. Thus, all activities are organized and centered on the teacher/lecturer and the students receive the knowledge passively. The students’ reasoning and knowledge development as far as the teacher/lecturer’s. This causes the students’ activities to be limited and makes the students unable to increase their learning optimally. Based on the explanation above it is clear that PBL model is better than conventional teaching model since the former engages all senses of the students in the teaching and learning process. As the consequence, the learning achievement in the form of ability to understand concepts and ability to solve mathematical problems of the students who learn through PBL model are better than those who learn through conventional teaching model (Santyasa, 2014: 31).

The result of hypothesis test shows that there is a significant understanding of the concept and ability of problem-solving of mathematics between students who are assessed using authentic assessment model with the conventional assessment. Significantly the authentic assessment affects the understanding of concepts and the ability to solve mathematical problems. Students were assessed by using authentic assessment are superior to those assessed using conventional assessment. These results support the results of the study: 1) Arhin (2015: 109), that authentic assessment learning improves the ability to solve mathematical problems, students' self-confidence in mathematics increases as they feel more competent) and 2) Kinay & Birsen (2016: 51), the authentic appraisal approach affects the problem-solving skills of prospective teachers.

Authentic assessment emphasizes the ability of students to demonstrate their knowledge in a meaningful and tangible way. In addition, students are required to demonstrate knowledge, skills, and strategies by creating responses or products that are backed by theoretical knowledge. So that students feel that the tasks they do really meaningful and they immediately know the level of knowledge of a problem. Furthermore, through self-assessment done at the end of learning, they can honestly see the advantages and disadvantages, for further, this deficiency becomes the goal of improvement. As a result, the students' responsibilities towards the process and the achievement of their learning goals are increasing, and ultimately their learning achievement tends to increase.

The application of authentic assessment of classroom learning provides a good psychological impact on students. Student involvement in the assessment process can lead to social interactions and mutual trust between one student and another and the interaction between students and lecturers. The perception of students that with their involvement in assessment activities, in general, has increased their confidence, responsibility, and motivation. Social interaction can occur through mutual control between the assessment conducted by students and the lecturers to the students' self-assessment, resulting in a valid, objective, and acceptable assessment by both parties ie students and lecturers.

Conventional assessment is often referred to by the term paper and pencil assessment. Implementation, separate from the student learning process. Feeding back to college students is too late and almost impossible to do so that learning improvement objectives as a result of assessment are not immediately possible and even uncertain for the purpose of the meeting (Depdiknas, 2009). This assessment model also does not provide information to students during the learning process. Students do not know the parts of the subject matter that have been understood and that have not been understood. As a result, students can not make efforts to improve their learning. The learning outcomes he earned were less than satisfactory. Theoretically, that authentic assessment models tend to be superior to conventional assessment models in achieving conceptual understanding as well as solving math problems.

Significantly, there is an interaction effect between the learning model with the assessment of the ability of concept comprehension and mathematical problem-solving. The influence of PBL model on the achievement of conceptual understanding and problem-solving ability is influenced by the applied assessment. These interactions have a positive impact on learning, among others, as follows.

The average score of concept comprehension and math problem-solving ability of the student group is facilitated by PBL with the highest authentic assessment among other groups. The average student group score is facilitated by conventional learning with the lowest conventional assessment. While the average score of the student group is facilitated by conventional learning with authentic assessment is better than the group of students who facilitated the model of PBL with the conventional assessment.

Authentic assessment situations require students to demonstrate knowledge, skills, and strategies by creating responses or products backed by theoretical knowledge. Students engage in learning, while doing an assessment so

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that they know how to think. Activity and motivation of students in following learning increases and can always assess themselves so know the level of mastery of the material being studied. They do self-reflection on the material being studied. Next can know its own weaknesses and know the parts of the material that has not been understood. Strategies for improving learning efforts can be determined appropriately. They change from passive to highly active and skilled at solving problems. A positive attitude and good skills can boost confidence and encourage increased motivation to learn. In the end, the student learning outcomes would be very satisfying.

4. Conclusion

The results of the analysis concluded: 1) there is a difference in mathematical concept understanding and ability to solve mathematical problems between the students facilitated by PBL and those facilitated by conventional teaching model; 2) there is a difference in ability to understand mathematical concepts between the students facilitated by authentic assessment and those facilitated by conventional assessment model, and 3) there is an interaction effect between teaching model and assessment on the ability to understand mathematical concepts and ability to solve mathematical problems.

Implications, among others, PBL is one of the innovative learning models that can be used as one of the efforts to improve the ability to understand the concept and solving math problems. Another factor that also has a positive effect on student learning outcomes, namely the assessment applied by lecturers in learning. Therefore, the lecturer of mathematics should be able to choose the appropriate assessment as a means to control during the process of learning activities. Authentic assessment can be selected as an appropriate assessment in the PBL Authentic assessment can be one means to improve the effectiveness of the learning model. Because authentic assessment is an actual and integrated assessment during the learning process. The selection and application of the correct authentic assessment can certainly improve the quality of student learning outcomes. PBL collaborated with authentic assessment can be referred to as an effort to improve understanding of concepts and problem-solving abilities of mathematics. PBL and authentic assessment should be broadly implemented, not limited to mathematics course but also in other courses either in the class/laboratory.

Suggestions: 1) PBL and authentic assessment need to be further developed as an alternative learning model and assessment in mathematics learning, 2) advanced research related to PBM and authentic assessment on mathematical materials whose characteristics are appropriately taught using PBL and assessed using authentic assessment by involving samples more broadly, and 3) the selection of the problem becomes very important in the PBL, the problem needs to be chosen so that it challenges the student's interest in completing it, connecting with previous experience and learning, and requiring cooperation and strategies to solve it. In addition, there should also be an in-depth study of learning tools that support PBL implementation and authentic assessment in the classroom or in the laboratory.

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Statement of authorship
The authors have a responsibility for the conception and design of the study. The authors have approved the final article.

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References


## Biography of Authors

<table>
<thead>
<tr>
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<th>Education and Experience</th>
</tr>
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<tr>
<td><strong>I Ketut Darma</strong></td>
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