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On Moderate Fuzzy Analytic Hierarchy Process Pairwise Comparison Model With Sub-criteria

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Abstract

Decisions usually involve getting the best solution, selecting the suitable experiments, most appropriate judgments, taking the quality results etc., using some techniques. Every decision making can be considered as the choice from the set of alternatives based on a set of criteria. The fuzzy analytic hierarchy process is a multi-criteria decision making and is dealing with decision-making problems through pairwise comparisons mode. The weight vectors from this comparison model are obtained by using the extent analysis method. This paper concern with an alternate method of finding the weight vectors from the original fuzzy AHP decision model (moderate fuzzy AHP model), that has the same rank as obtained in original fuzzy AHP and ideal fuzzy AHP decision models.

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1. Introduction

The analytical hierarchy process (AHP) was developed by Saaty in 1971. This process is used to find the weight vectors for decision-making problems in an uncertain situation from the pairwise comparison model with multiple criteria and alternatives. The function of AHP is to systemize complex and unstructured problems, which it resolves hierarchically from the higher levels to lower levels. Through quantitative judgment, AHP simplifies the decision making processes that relied on intuition to obtain the weight of the alternatives corresponding to the criteria or alternative corresponding sub-criteria and sub-criteria with respect to main criteria and this provides the sufficient information for decision-makers. Alternatives with criteria having greater weight give the higher weight. The AHP performs problem analysis, which can reduce the risk of mistakes in decision making. However, AHP use cannot

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overcome the subjectivity, inaccuracy, and fuzziness produced when making decisions. So, by introducing and applying fuzzy set theory and fuzzy operation on AHP, which can ameliorate these failures.

Since basic AHP does not include vagueness for personal judgments, it has been improved by benefiting from the fuzzy logic approach. In Fuzzy AHP, the pairwise comparisons of both criteria and the alternatives are performed through the linguistic variables, which are represented by triangular numbers. If the uncertainty (fuzziness) of human decision making is not taken into account, the results from the models can be misleading. The fuzzy theory has been applied in a variety of fields since its introduction. Fuzzy AHP methods are proposed to solve various types of problems. The main theme of these methods is using the concepts of fuzzy set theory and hierarchical structure analysis to present systematic approaches in selecting or justifying alternatives. In this study, the extent analysis method by Chang (1992, 1996) is adopted because the steps of this approach are relatively easier, less time taking and less computational expense than many other fuzzy AHP approaches.

Fuzzy set theory was first introduced by Zadeh in 1965; it emphasizes the fuzziness of human thinking, reasoning, and cognition of surroundings. A number of conventional quantitative analysis methods cannot analyze such things efficiently. Furthermore, fuzzy logic can analyze the ambiguity and vagueness of the decision-making problem. Fuzzy logic is a method to formalize the human capacity of imprecise or approximate reasoning. Such reasoning represents the human ability reason approximately and judges under uncertainty. In fuzzy logic, all truth is partial or approximate. In this sense, this reasoning has been termed as interpolative reasoning, where the process of interpolating between the binary extremes of true and false is represented by the ability of fuzzy logic to encapsulate partial truths. The fuzzy set can be defined as follows.

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in U)\}$$

Where $\tilde{A}$ is a fuzzy set? $\mu_{\tilde{A}}(x)$ is called the membership function. U is the universe of discourse. $\mu_{\tilde{A}}(x)$ ranges between 0 and 1. This is called the degree of membership. The fuzzy set can better describe the characteristics of things compared to conventional binary logic. In conventional crisp sets, the value of the membership function can only be 0 or 1.

A Triangular fuzzy Number is a special case of fuzzy number. It is defined by a triplet $\tilde{A}$ = (a, b, c). This representation is interpreted as a membership function

$\mu_{\tilde{A}} : \mathbf{R} \to [0, 1]$ as follows.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{if } x < a \\ \dfrac{x-a}{b-a} & \text{if } a < x < b \\ \dfrac{c-x}{c-b} & \text{if } b < x < c \\ 0 & \text{if } x > x \end{cases}$$

Algebraic Operations: Let $\tilde{A} = (a_1, b_1, c_1)$ and

$\tilde{B} = (a_2, b_2, c_2)$ be two triangular fuzzy numbers.

(i) Addition of Triangular Fuzzy Numbers $\oplus$:

$$\tilde{A} \oplus \tilde{B} = (a_1 + a_2, b_1+b_2, c_1+c_2)$$

(ii) Multiplication of Triangular Fuzzy Numbers$\otimes$:

$$\tilde{A} \otimes \tilde{B} = (a_1a_2, b_1b_2, c_1c_2);\ a_1 > 0, a_2 > 0$$

(iii) Division of Triangular Fuzzy Number $\varnothing$:

$$\tilde{A} \varnothing \tilde{B} = \left(\frac{a_1}{c_2}, \frac{b_1}{b_2}, \frac{c_1}{a_2}\right);\ a_1 > 0,\ a_2 > 0$$

(iv) Inverse of a Triangular Fuzzy Number:

$$\tilde{A}^{-1} = (a_1, b_1, c_1)^{-1} = \left(\frac{1}{c_1}, \frac{1}{b_1}, \frac{1}{a_1}\right);\ a_1 > 0$$

A Triangular Fuzzy Number Matrix of order n x m is defined as A = $\left(\tilde{a}_{ij}\right)_{n \times m}$ where $\tilde{a}_{ij}$ is a triangular fuzzy number.

The two sets, X = {x_1, x_2, x_3, …,x_n} as an object set, and G ={u_1, u_2, u_3,….,u_m} as a goal set, can be defined in initial stage. According to the principles of Chang's [3] extent analysis, each object is considered correspondingly, and extent analysis for each of the goal, g_i is executed. It means that it is possible to obtain the values of m extent analyses that can be demonstrated as $M_{g_i}^1, M_{g_i}^2, \ldots, M_{g_i}^m$ i=1,2,…,n, where $M_{g_i}^j$ (j=1,2,…,m) are triangular fuzzy numbers. After identifying initial assumptions, Chang's extent analyses [3], [8], [9] can be examined in four main steps:

Step 1: The value of fuzzy synthetic extent with respect to the object is represented as,

$F_i = \sum_{j=1}^{m} M_{g_i}^j \otimes \left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{g_i}^j\right]^{-1}$, and fuzzy addition operation of m extent analysis values can be performed for particular matrix such that $\sum_{j=1}^{m} M_{g_i}^j = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j\right)$. Then, the fuzzy addition operation of $M_{g_i}^j$ (j=1,2,…., m) values such that $\sum_{i=1}^{n}\sum_{j=1}^{m} M_{g_i}^j = \left(\sum_{i=1}^{n} l_i, \sum_{i=1}^{n} m_i, \sum_{i=1}^{n} u_i\right)$ are performed to obtain $\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^j\right]^{-1}$. At the end of the Step 1, the inverse of the determined vector can be expressed as follows.

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{g_i}^j\right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i}\right)$$

Step 2 : The degree of possibility of $M_1=(l_1,m_1,u_1)> M_2 = (l_2,m_2,u_2)$ is defined as $D(M_1>M_2)= \sup_{x\geq y} [\min (\mu_{M1}(x), \mu_{M2}(x))]$,

When a pair (x, y) exists such that x>y and $\mu_{M_1}(x) = \mu_{M_2}(x)$, then we have $D(M_1>M_2) = 1$.

Since M_1 and M_2 are convex fuzzy numbers we have that

$D(M_1 > M_2) = 1$ if f $m_1 > m_2$

$D(M_2 > M_1) = hgt(M_1 \cap M_2) = \mu_{M_1}(d)$

Where d is the ordinate of the highest intersection point between $\mu_{M_1}(d)$ and $\mu_{M_2}(d)$. Also the above equation can be equivalently expressed as follows.

$D(M_2 > M_1) = hgt(M_1 \cap M_2) = \mu_{M_1}(d)$

$$= \begin{cases} 1, & \text{if } m_2 > m_1, \\ 0, & \text{if } l_1 > u_2, \\ \dfrac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)} & \text{Otherwise,} \end{cases}$$

Step 3: From obtaining k(k=1, 2, ….n) convex fuzzy numbers, the degree possibility for a i^{th} convex fuzzy number to be greater than k convex fuzzy numbers M_i (i =1, 2, ….,k) can be defined as follows.

$D(F_i > F_k) = D(F_i > F_1)$ and $D(F_i > F_2) \ldots D(F_i > F_k)$

$= D(F_i > F_1,F_2, F_3,\ldots F_k)$ with $i \neq k$.

$d'(A_i)=\min [D(F_i > F_1,F_2, F_3,\ldots F_k)]$ with $i \neq k$.

$$W' = (d'(A_1), d'(A_2), \ldots, d'(A_n))^T$$

where A_i (i =1, 2, …, n) are n elements.

Step 4: Via normalization, the normalized weight vectors are $W= (d(A_1),d(A_2)\ldots,d(A_n))^T$, where W is a nonfuzzy number that gives weight vectors of an attribute or an alternative over other. Thus we get the original fuzzy AHP decision model with weight vector *W*.

Step 5: Form the original Fuzzy AHP decision matrix, multiply the weight vectors of the main criteria with corresponding weight vectors of the sub-criteria to get resulting criteria weights. Multiply these with corresponding priority Vectors of Alternatives. The sum of these values is the final priority Vector for a respective alternative. In such a way, we find the final priority vectors for the remaining alternatives.

Step 6: Also we can get the ideal Fuzzy AHP decision Matrix, by dividing the entries in the column of the original Fuzzy AHP matrix for the corresponding criterion with the largest entry in that particular column. Multiply these values of the alternatives with corresponding the resulting criterion weights. Sum these Values to get the final priority vector for the respective alternative. In such a way we find the final priority vectors for the remaining alternatives. After normalizing the final priority Vectors, to have the values with ranking.

Step 7: It can be extended to find the final alternative priority vectors for all alternatives from the original Fuzzy AHP decision matrix. It can be obtained from the following formula [7], [8]

$$MS_i = \sum_{j=1}^{m} W_j (W_j + S'_{ij})$$

Where W_j is the weight vector for corresponding resulting criteria weight and S'_{ij} is the weight vector of the ith alternative and jth resulting criterion of the original Fuzzy AHP decision matrix. We get a moderate Fuzzy AHP decision matrix.

After normalization, we have ranked the alternatives. Finally, we have the same ranking for original Fuzzy AHP decision matrix, Ideal Fuzzy AHP decision matrix, and moderate Fuzzy AHP decision matrix, even though different values of the final priority vectors of respective alternatives for these 3 methods.

2. Materials and Methods

Geometrical Interpretation

The structure of the typical problem can consist of Criteria, sub-criteria with respect to criteria and the alternatives with respect to the sub-criteria. Each alternative can be evaluated in terms of the sub-criteria with the main criteria and the relative importance of each criterion can be estimated as well. Suitable performance values for criteria, subcriteria, and alternatives are given. The problem has a three-level hierarchy of alternatives and criteria.

3. Results and Discussions

Model of the Problem

Suppose the expert has to choose a company for excellent service. Three main Criteria have been chosen for evaluation of alternative with better service namely Quantity. Quality and time. Each main criterion is divided into two sub-criteria, namely purchasing and production for quantity, higher productive and lower productive for quality and probabilistic and deterministic for time. Three alternative companies have been chosen for manufacturing. Our goal is to select a company in order to satisfy all the criteria in the best way.

The solution is based on the proposed fuzzy AHP method. The procedure in applying the fuzzy AHP is to construct three-level hierarchy of alternative, sub-criteria and main criteria as shown in figure 1.

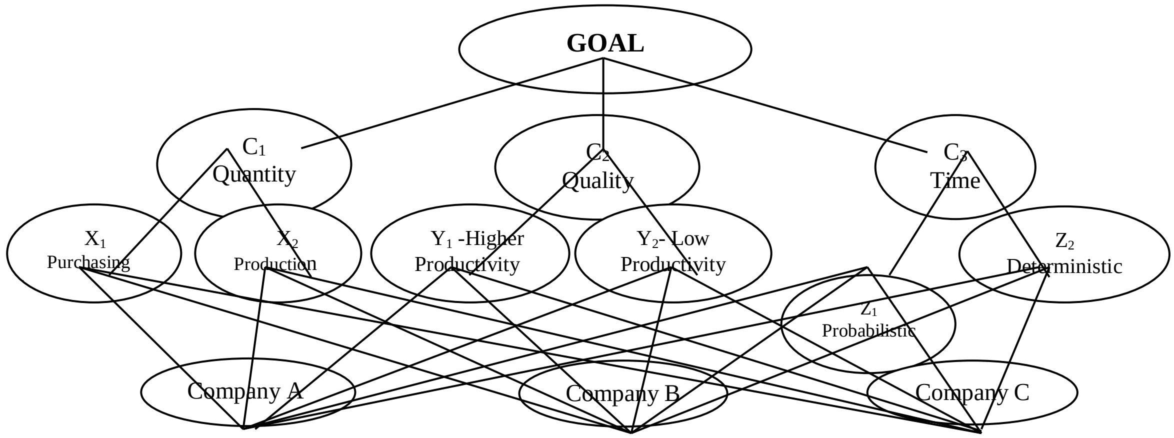

Figure 1. Sub criteria and main criteria

To decide the final Priority of different Criteria, a triangular fuzzy number is used in pairwise comparison and the extent analysis method for the synthetic value of the pairwise comparison is applied.

The evaluation of the fuzzy scale and their definition used by the experts are in table 2.

Table 2
Fuzzy AHP Scale

	Definition	Triangular fuzzy number	Reciprocal Fuzzy number
1	Equally importance	(1,1,1)	(1,1,1)
2	Weakly importance	(1,1,3)	(1/3,1,1)
3	Essentially importance	(1,2,3)	(1/3,1/2,1)
4	Moderately importance	(1,3,3)	(1/3,1/3,1)
5	Strongly importance	(1,3,5)	(1/5,1/3,1)
6	Very strongly importance	(3,5,7)	(1/7,1/5,1/3)
7	Extremely impotance	(5,7,9)	(1/9,1/7,1/5)

Table 2
Fuzzy pairwise comparison of the main criteria

Goal	C_1	C_2	C_3
C_1	(1,1,1)	(1,1,3)	(1,3,5)
C_2	(1/3,1,1)	(1,1,1)	(1/7,1/5,1/3)
C_3	(1/5,1/3,1)	(3,5,7)	(1,1,1)

∴ The normalized weight vector for the main criteria is calculated as

$$W_C = (0.433, 0.082, 0.485) \quad (1)$$

Table 3
Fuzzy pairwise comparison of the subcriteria with respect to C_1

C_1	X_1	X_2
X_1	(1,1,1)	(1,2,3)
X_2	(1/3,1/2,1)	(1,1,1)

The normalized weight vectors are

W_{C_1} = (0.693,0.307) (2)

Table 4
Fuzzy pair wise comparison of the subcriteria with respect to C_2

C_2	Y_1	Y_2
Y_1	(1,1,1)	(1,3,5)
Y_2	(1/5,1/3,1)	(1,1,1)

The normalized weight vector is

Wc_2 = (0.833 , 0.167) (3)

Table 5
Fuzzy pairwise comparison of the subcriteria with respect to C_3

C_3	Z_1	Z_2
Z_1	(1,1,1)	(1,3,3)
Z_2	(1/3,1/3,1)	(1,1,1)

The normalized weight vector is

Wc_3 = (0.742 , 0.258)

(4)

Table 6
Fuzzy pairwise comparison model of alternatives with respect to X_1

X_1	A	B	C
A	(1,1,1)	(3,5,7)	(1/3,1/3,1)
B	(1/7,1/5,1/3)	(1,1,1)	(1,3,5)
C	(1,3,3)	(1/5,1/3,1)	(1,1,1)

The normalized weights vector for the alternative are calculated as

W_{X_1} = $(0.398, 0.312, 0.290)^T$ (5)

Table 7
Fuzzy pairwise comparison model of the alternatives with respect to X_2

X_2	A	B	C
A	(1,1,1)	(5,7,9)	(1/3,1/2,1)
B	(1/9,1/7,1/5)	(1,1,1)	(1,3,5)
C	(1,2,3)	(1/5,1/3,1)	(1,1,1)

The normalized weights vector for the alternative with respect to X_2

W'_{X_2} = $(0.530, 0.276, 0.193)^T$ (6)

Table 8
Fuzzy pairwise comparison model of alternatives with respect to Y_1

Y_1	A	B	C
A	(1,1,1)	(1/7,1/5,1/3)	(5,7,9)
B	(3,5,7)	(1,1,1)	(1/5,1/3,1)
C	(1/9,1/7,1/5)	(1,3,5)	(1,1,1)

The normalized weights vector for the alternative with respect to Y_1

W_{Y_1} = $(0.422, 0.349, 0.229)^T$ (7)

Table 9
Fuzzy pair wise comparison model of alternatives with respect to Y_2

Y_2	A	B	C
A	(1,1,1)	(1,3,5)	(1/9,1/7,1/5)
B	(1/5,1/3,1)	(1,1,1)	(1,1,3)
C	(5,7,9)	(1/3,1,1)	(1,1,1)

The normalized weights vector for the alternative with respect to Y_2

$$W_{Y_2} = (0.272, 0.168, 0.560)^T \tag{8}$$

Table 10
Fuzzy pairwise comparison model of alternatives with respect to Z_1

Z_1	A	B	C
A	(1,1,1)	(1/3,1,1)	(1,3,5)
B	(1,1,3)	(1,1,1)	(1/9,1/7,1/5)
C	(1/5,1/3,1)	(5,7,9)	(1,1,1)

The normalized weights vector for the alternative with respect to Z_1

$$W_{Z_1} = (0.342, 0.121, 0.537)^T \tag{9}$$

Table 11
Fuzzy pairwise comparison model of alternatives with respect to Z_2

Z2	A	B	C
A	(1,1,1)	(1,3,3)	(1/3,1,1)
B	(1/3,1/3,1)	(1,1,1)	(5,7,9)
C	(1,1,3)	(1/9,1/7,1/5)	(1,1,1)

The normalized weights vector for the alternative with respect to Z_2

$$W_{Z_2} = (0.262, 0.637, 0.101)^T \tag{10}$$

Table 12
Fuzzy AHP Decision Model

Main Criteria	C_1		C_2		C_3	
	0.433		0.082		0.485	
Subcriteria	X_1	X_2	Y_1	Y_2	Z_1	Z_2
	0.693	0.307	0.833	0.167	0.742	0.258
A	0.398	0.530	0.422	0.272	0.342	0.262
B	0.312	0.276	0.349	0.168	0.121	0.637
C	0.290	0.193	0.229	0.560	0.537	0.101

Table 13
Original Fuzzy AHP Decision Model

Main Criteria	X_1	X_2	Y_1	Y_2	Z_1	Z_2	Final Priority Vector	Rank
Sub criteria	0.300	0.133	0.068	0.014	0.360	0.125		
A	0.398	0.530	0.422	0.272	0.342	0.262	0.378	1
B	0.312	0.270	0.349	0.168	0.121	0.637	0.280	3
C	0.290	0.193	0.229	0.560	0.537	0.101	0.343	2

Table 14
Ideal Fuzzy AHP Decision Model

Main Criteria	X_1	X_2	Y_1	Y_2	Z_1	Z_2	Final Priority Vector	Normalization	Rank
Subcriteria	0.300	0.133	0.068	0.014	0.360	0.125			
A	1.000	1.000	1.000	0.500	0.637	0.413	0.789	0.383	1
B	0.790	0.509	0.828	0.250	0.228	1.000	0.572	0.278	3
C	0.731	0.364	0.552	1.000	1.000	0.163	0.699	0.339	2

Table 15
Moderate Fuzzy AHP decision model

Main Criteria	X_1	X_2	Y_1	Y_2	Z_1	Z_2	Final Priority Vector	Normalization	Rank
Subcriteria	0.300	0.133	0.068	0.014	0.360	0.125			
A	0.209	0.088	0.033	0.004	0.253	0.048	0.635	0.359	1
B	0.184	0.054	0.028	0.003	0.173	0.095	0.537	0.303	3
C	0.177	0.043	0.020	0.008	0.323	0.028	0.599	0.338	2

Therefore, the best selection is A followed by C and C is followed by B. Hence, company A is the best performance in order to satisfy all criteria. Finally, we observe that the original Fuzzy AHP, the ideal Fuzzy AHP, and the moderate Fuzzy AHP decision matrices have the same ranking for the said 3 alternatives, even though they assigned different final priority vectors for these alternatives.

4. Conclusion

The fuzzy AHP is used for ranking with weight vectors of pairwise comparison matrices. It provides an effective solution for solving the MCDM problem. We can involve any relative importance of criteria and that of alternatives in the moderate fuzzy AHP. Also, moderate fuzzy AHP allows for a sensitivity analysis in term of the relative priorities, by adjusting the ranking values. Application of the moderate fuzzy AHP of the MCDM can be discussed in further research proposals. The numerical problem shows the proposed fuzzy analysis and its applicability in providing a valuable decision support.

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Statement of authorship
The author(s) have a responsibility for the conception and design of the study. The author(s) have approved the final article.

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