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Continuous Time H-Infinity Filter with Asymptotic Convergence

Swetha Amit [a]
Raol Jitendra [b]

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Abstract

In this paper H-infinity, a posteriori filter (HIPF) is converted to a continuous time H-infinity (HI) filter. Then, an observer is presented which uses the gain and the state error Gramian from the continuous time HI filter (CTHF). Asymptotic stability result of the observer's error dynamics is derived using the Lyapunov energy functional. The performance of the CTHF is evaluated using numerical simulations carried out in MATLAB. These results establish the local stability of the underlying CTHF. This type of result is novel in the literature on HI theory of filters and the observers.

Author correspondence:
Swetha Amit,
Asstt. Prof., Dept. of Telecommunications Engineering, Ramaiah Institute of Technology, and Research Scholar, Jain University, Bangalore, India
Email address: *swetha.amit@gmail.com*

1. Introduction

Conventionally Kalman filter and its variants are used for estimation of states of a dynamic system from its input output data; especially using the noisy measurements (Raol, 2004). This process requires assumptions on the statistics of the process noise and measurement noise (Raol, 2017). Often such information is not available. H-infinity filtering theory has provided an alternative paradigm in a deterministic framework for the estimator of the states of a dynamic system with minimum assumptions (Hassibi, 1996). Another alternative is the deterministic domain is the theory of observers. In this paper first HI based a posteriori filter is briefly described. Then, this discrete time filter (HIPF) is converted into continuous time HI based filter (CTHF). Interestingly, the resulting CTHF is similar to the HIPF with some additional terms. Then, an observer is presented that utilizes the gain and the state error Gramian from this CTHF. Using Lyapunov energy functional, asymptotic stability result of the observer's error dynamics is derived. The performance of the CTHF is evaluated using MATLAB based numerical simulations. These results establish the asymptotic stability of the underlying CTHF. The result of the present study is a novel contribution to the literature of HI theory of filters and the observers.

[a] Jain University, Bangalore, India
[b] Ramaiah Institute of Technology, Bangalore

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2. Materials and Methods

The research has been based on the mathematical derivation of continuous time HI filter (CTHF). The basic HI filter is based on the minimization of the H-infinity norm and is a robust algorithm. Hence, H-infinity a posteriori filter (HIPF) is converted to a continuous time H-infinity (HI) filter. The continuous time development offers good flexibility to study analytical results. Then, an observer is proposed based on the CTHF, and asymptotic stability result of the observer's error dynamics is derived using the Lyapunov energy functional. It is established that the error dynamics of the nonlinear observer with the gain and state error 'covariance' Gramian from the CTHF are locally asymptotically stable, and interestingly this result also establishes that the CTHF algorithm would be asymptotically stable. The latter is true because the asymptotic convergence result is based on the observer error dynamics that use the CTHF gain and the Gramian matrix from the CTHF. This is a novel interpretation in this paper. The performance of the CTHF is evaluated using numerical simulations carried out in MATLAB.

3. Results and Discussions

3.1 H-Infinity a Posteriori Filter

Consider a linear discrete time dynamic system

$$x(k+1) = \phi x(k) + Gw(k) \tag{1}$$
$$z(k) = Hx(k) + v(k) \tag{2}$$

In (1), and (2), the variables have usual meanings from control and system theory. The KF is a very well known and a popular filtering algorithm for estimation of the states, x(.), utilizing the measurements z(.). However, it requires statistical assumptions on the process noise w(.), and the measurement noise v(.). There is an alternative theory based on the H-infinity norm, that has resulted into several HI based filtering and control algorithms (Gelb, 1974). This HI filter is based on the minimization of the H-infinity norm, and it is supposed to be a robust algorithm. The HIPF algorithm is given here. First, the (covariance) state error Gramian (SEG) propagation is obtained as

$$P(k+1) = \phi P(k)\phi^T + \phi P(k-1)\phi^T + Q - \phi P(k)[H^T \quad L^T]R_e^{-1}\begin{bmatrix} H \\ L \end{bmatrix} P(k)\phi^T \tag{3}$$

In (3), the composite measurement (covariance) Gramian matrix is obtained as

$$R_e = \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} H \\ L \end{bmatrix} P(k)[H^T \quad L^T] \tag{4}$$

In (4), γ is a factor that specifies the upper bound on the energy (variance) gain from the input energies due to the disturbances (w(.), and v(.)), and the input error in the state initial condition to the output state error energy. The HIPF filter gain is obtained as

$$K = P(k+1)H^T(HP(k+1)H^T + I)^{-1} \tag{5}$$

The measurements/data update of the state estimate is given as

$$\hat{x}(k+1) = \tilde{x}(k) + K(z(k+1) - H\tilde{x}(k)) \tag{6}$$

In (6), the previous state estimate is obtained as $\tilde{x}(k) = \phi\hat{x}(k)$ (7)

In the context of HI/HIPF theory, all the variables are considered as generalized 'random' variables, and the associated 'covariance' like matrices are called Gramians.

3.2 Continuous time H-Infinity filter (CTHF)

The continuous time development has often a very good flexibility to study analytical results. Hence, the HIPF is converted here to continuous time filter. The continuous time linear dynamic system is given as

$$\dot{x}(t) = Ax(t) + w(t) \tag{8}$$

$$z(t) = Hx(t) + v(t) \tag{9}$$

In (8) and (9), the variables have usual meanings. The main idea here is to convert the HIPF to continuous time filter, and hence, we utilize the following substitutions, as a first order approximation (Raol *et al.*, 2004):

$$\phi(k) = I + A\Delta t \tag{10}$$

$$Q(k) = GQG^T \Delta t\,; and\; R(k) = \frac{R}{\Delta t} \tag{11}$$

The equivalences of (10), and (11) are used to derive the continuous time HI filter from the discrete time HIPF. The approach is to use the differential equations of the HIPF and then observe their behavior as the discrete time step $\Delta t \to 0$.

First, the HIPF gain is handled as follows:

$$\frac{1}{\Delta t} K(k) = \frac{1}{\Delta t} P(k+1) H^T (HP(k+1)H^T + I)^{-1} \tag{12}$$

$$\frac{1}{\Delta t} K(k) = P(k+1) H^T (\Delta t HP(k+1)H^T + I\Delta t)^{-1}$$

$$\frac{1}{\Delta t} K(k) = P(k+1) H^T (\Delta t HP(k+1)H^T + R)^{-1};\, R = I\Delta t \tag{13}$$

In the limit as the $\Delta t \to 0$, the continuous time HI gain is obtained from (13) as

$$\lim_{\Delta t \to 0} \frac{1}{\Delta t} K(k) = \lim_{\Delta t \to 0} P(k+1) H^T (\Delta t HP(k+1)H^T + R)^{-1} \tag{14}$$

$$K(t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} K(k) = P(t) H^T R^{-1} \text{ ; wherein R(t) is given as } R = I\Delta t \tag{15}$$

Next, (10) is substituted in (3) to obtain

$$P(k+1) = (I + A\Delta t) P(k) (I + A\Delta t)^T + GQG^T \Delta t - (I + A\Delta t) P(k) [H^T \quad L^T] R_e^{-1} \begin{bmatrix} H \\ L \end{bmatrix} P(k) (I + A\Delta t)^T \tag{16}$$

Simplifying (16) at first stage one gets, neglecting higher product term,

$$P(k+1) = P(k) + AP(k)\Delta t + P(k)A^T \Delta t + GQG^T \Delta t - (I + A\Delta t)[P(k)H^T \quad P(k)L^T] R_e^{-1} \begin{bmatrix} HP(k) \\ LP(k) \end{bmatrix} (I + A\Delta t)^T \tag{17}$$

Now, the gains are specified as

$$K = PH^T R^{-1};\, HP = R^T K^T \tag{18}$$

$$K_l = PL^T;\, LP = K_l^T \tag{19}$$

Substituting (18), and (19) in (17), one obtains

$$P(k+1) = P(k) + AP(k)\Delta t + P(k)A^T \Delta t + GQG^T \Delta t - (I + A\Delta t)[KR \quad K_l] R_e^{-1} \begin{bmatrix} R^T K^T \\ K_l^T \end{bmatrix} (I + A\Delta t)^T \tag{20}$$

Next, the central part of the last term is written as follows for simplicity

$$P_{cd} = [KR \quad K_l] R_e^{-1} \begin{bmatrix} R^T K^T \\ K_l^T \end{bmatrix} \tag{21}$$

Substituting (21) in (20), and then simplifying one obtains, neglecting higher product term,

$$P(k+1) = P(k) + AP(k)\Delta t + P(k)A^T \Delta t + GQG^T \Delta t - (I + A\Delta t) P_{cd} \,(I + A\Delta t)^T \tag{22}$$

$$P(k+1) = P(k) + AP(k)\Delta t + P(k)A^T \Delta t + GQG^T \Delta t - P_{cd} - AP_{cd}\Delta t - P_{cd}A^T \Delta t \tag{23}$$

Now, forming the differential from (23), and dividing both the sides by Δt one obtains

$$\frac{P(k+1) - P(k)}{\Delta t} = \frac{1}{\Delta t} [\{AP(k) + P(k)A^T\}\Delta t + GQG^T \Delta t - P_{cd} - \{AP_{cd} + P_{cd}A^T\}\Delta t] \tag{24}$$

Then, applying the limit $\Delta t \to 0$, one obtains

$$\lim_{\Delta t \to 0} \frac{P(k+1) - P(k)}{\Delta t} = \lim_{\Delta t \to 0} \frac{1}{\Delta t} [\{AP(k) + P(k)A^T\}\Delta t + GQG^T \Delta t - P_{cd} - \{AP_{cd} + P_{cd}A^T\}\Delta t] \tag{25}$$

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$$\dot{P}(t) = AP(k) + P(k)A^T + GQG^T - \lim_{\Delta t \to 0} \frac{1}{\Delta t} P_{cd} - AP_{cd} - P_{cd}A^T \quad (26)$$

Next, the limit of the fourth term is evaluated as follows:

$$\lim_{\Delta t \to 0} \frac{1}{\Delta t} P_{cd} = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \left\{ [KR \quad K_l] R_e^{-1} \begin{bmatrix} R^T K^T \\ K_l^T \end{bmatrix}^T \right\} \quad (27)$$

$$\lim_{\Delta t \to 0} \frac{1}{\Delta t} P_{cd} = [KR \quad K_l] \left\{ \lim_{\Delta t \to 0} \frac{1}{\Delta t} (R_e)^{-1} \right\} \begin{bmatrix} R^T K^T \\ K_l^T \end{bmatrix}^T \quad (28)$$

The limit of the bracketed term is evaluated as follows:

$$\lim_{\Delta t \to 0} \frac{1}{\Delta t} (R_e)^{-1} = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \left\{ \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} H \\ L \end{bmatrix} P(k) [H^T \quad L^T] \right\}^{-1} \quad (29)$$

$$= \lim_{\Delta t \to 0} \left\{ \begin{bmatrix} I\Delta t & 0 \\ 0 & -\gamma^2 I \Delta t \end{bmatrix} + \begin{bmatrix} H \\ L \end{bmatrix} P(k) [H^T \quad L^T] \Delta t \right\}^{-1} \quad (30)$$

Evaluating the limit in (30), one obtains the following

$$R_{ec}^{-1} = \begin{bmatrix} I\Delta t & 0 \\ 0 & -\gamma^2 I \Delta t \end{bmatrix}^{-1} \quad (31)$$

Substituting (31) in (28), one gets

$$P_c = [KR \quad K_l] (R_{ec})^{-1} \begin{bmatrix} R^T K^T \\ K_l^T \end{bmatrix}^T \quad (32)$$

Substituting (32) in (26) the final equation for the time propagation of the state error Gramian matrix is obtained as

$$\dot{P}(t) = AP(t) + P(t)A^T + GQG^T - P_c - AP_{cd} - P_{cd}A^T \quad (33)$$

In a similar manner, one can obtain the continuous time state estimation equation as follows

$$\hat{x}(k+1) = \phi\hat{x}(k) + K(z(k+1) - H\phi\hat{x}(k)) \quad (34)$$

Substituting (10) in (34), taking the limit as $\Delta t \to 0$ and simplifying one obtains

$$\dot{\hat{x}} = A\hat{x} + PH^T R^{-1}(z - H\hat{x}) \quad (35)$$

3.3 The Continuous Time HI Filter-CTHF

Specify appropriate initial conditions: $x(0), P(0)$, etc.
The CTHF gain is given as

$$K(t) = P(t)H^T R^{-1}; \; R = I\Delta t; \; K_l(t) = P(t)L^T \quad (36)$$

The continuous time composite Gramian matrix is given as

$$P_c = [KR \quad K_l] (R_{ec})^{-1} \begin{bmatrix} R^T K^T \\ K_l^T \end{bmatrix};$$

$$R_{ec}^{-1} = \begin{bmatrix} I\Delta t & 0 \\ 0 & -\gamma^2 I \Delta t \end{bmatrix}^{-1} \quad (37)$$

The continuous time composite Gramian matrix with discrete measurement variance Gramian is given as

$$P_{cd} = [KR \quad K_l] R_e^{-1} \begin{bmatrix} R^T K^T \\ K_l^T \end{bmatrix}^T;$$

$$R_e = \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} H \\ L \end{bmatrix} P(k) [H^T \quad L^T] \quad (38)$$

It is important to note here that in the expression of $P_{cd}(.)$, (38), the discrete measurement 'variance' Gramian matrix continues (as was in the discrete time HIPF); however, in matrix $P_c(.)$, (37), the continuous time Gramian is used. The state error Gramian can be obtained by solving the following Riccati type matrix differential equation

$$\dot{P}(t) = AP(t) + P(t)A^T + GQG^T - P_c - AP_{cd} - P_{cd}A^T \tag{39}$$

$$\dot{\hat{x}} = A\hat{x} + PH^T R^{-1}(z - H\hat{x});$$

$$\dot{\hat{x}} = A\hat{x} + K(t)(z - H\hat{x}) \tag{40}$$

3.4 Observer based on Continuous time HI filter

Next, a continuous time observer that uses the gain from the CTHF is studied, and the asymptotic stability result is derived. A nonlinear continuous time system is given as

$$\dot{x} = f(x,t) \tag{41}$$

$$z = Hx \tag{42}$$

A nonlinear observer for the system of (41) can be given as

$$\dot{\hat{x}}(t) = f(\hat{x}(t),t) + L_o(t)\big(y(t) - \hat{y}(t)\big)$$

$$\hat{y}(t) = H\hat{x}(t) \tag{43}$$

In (43), $L_o(t)$ is observer gain matrix of appropriate dimension, and is taken from the CTHF as

$$L_o(t) = P(t)H^T R^{-1} \tag{44}$$

The matrix P(t) is obtained as the solution of the observer Riccati type differential (RTD) equation which is also directly based on the CTHF:

$$\dot{P}(t) = AP + PA^T + GQG^T - P_c - AP_{cd} - P_{cd}A^T \tag{45}$$

Interestingly (45) also needs (37), and (38). The required Jacobian for (45) is obtained as

$$A(t) = \frac{\delta f(.)}{\delta \hat{x}(t)} \tag{46}$$

By subtracting (43) from (41), the following observer error dynamics are obtained

$$\dot{e}(t) = A(t)e(t) - L_o(t)H\big(x(t) - \hat{x}(t)\big) + \phi(.)$$

$$= A(t)e(t) - L_o(t)He(t) + \phi(.) \tag{47}$$

In (47), new nonlinear function is

$$\phi(.) = -A(t)e(t) + f(x,t) - f(\hat{x},t) \tag{48}$$

In (48), the short forms for f and $\phi(.)$ can be used for simplicity

$$\phi(.) = \phi(x(t),\hat{x}(t),t); \quad f(x,t) = f(x(t),t) \tag{49}$$

The state errors are given as

$$e(t) = x(t) - \hat{x}(t) \tag{50}$$

3.5 Asymptotic stability result of the observer error dynamics

In order to study the local asymptotic behavior of the observer error dynamics of (47), it is necessary to consider the following conditions (Raff, 2006; Reif, 2017):

1. The solution of the matrix RTD equation (45) should be bounded

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$$p_l I \leq P(t) \leq p_u I \quad (51)$$

In (51), $p_l, p_u > 0$ are constants (for, $P(t)$ is theoretically, positive definite and symmetrical matrix), and are the lower and upper bounds respectively.

2. The nonlinearity (48) in the error dynamics should be bounded because the nonlinear functions should be bounded,

$$\|\phi(.)\| \leq \rho\|x(t) - \hat{x}(t)\|^2 \quad (52)$$

Then, the nonlinear observer error dynamics (47) are locally asymptotically stable, if basically the conditions 1 and 2 are satisfied; of course some more conditions are required, that would evolve as the derivation proceeds further. First, the LE functional is considered to establish the asymptotic stability of the error dynamics (47) as

$$V(t) = e^T(t)Y(t)e(t) \quad (53)$$

In (53), Y(t) is the normalizing matrix and is considered as an information matrix given as $Y(t) = P^{-1}(t)$. The matrix P(.) is called state error Gramian matrix, Y(.) as the information Gramian, and because the deterministic observer and the CTHF (in deterministic domain) are considered, the variables x(.), and y(.) are called the generalized 'random' variables. One can easily see that the LE functional is positive definite because of the condition 1, the inequality of (51):

$$\frac{1}{p_u}\|e(t)\|^2 \leq e^T(t)Y(t)e(t) \leq \frac{1}{p_l}\|e(t)\|^2 \quad (54)$$

The time derivative of the LE functional (53), under the constraints governed by error dynamics (47), gain (44), and the SEG (45) should be negative definite. This time derivative is obtained as

$$\dot{V}(t) = e^T(t)\dot{Y}(t)e(t) + e^T(t)Y(t)\dot{e}(t) + \dot{e}^T(t)Y(t)e(t) \quad (55)$$

Since, $\dot{Y} = -Y\dot{P}Y$, substituting this, and (45) in (55), the following expression is obtained

$$\dot{V}(t) = -e^T(t)Y(t)[AP + PA^T + GQG^T - P_c - AP_{cd} - P_{cd}A^T]Y(t)e(t) + e^T(t)Y(t)\dot{e}(t) + \dot{e}^T(t)Y(t)e(t) \quad (56)$$

Next, expression for error dynamics (47) is substituted in (56)

$$\dot{e}(t) = A(t)e(t) - L_o(t)He(t) + \phi(.) \quad (57)$$

$$\dot{V}(t) = -e^TY(t)[AP + PA^T + GQG^T - P_c - AP_{cd} - P_{cd}A^T]Y(t)e(t) + e^TY(t)\{Ae(t) - L_o(t)He(t) + \phi(.)\} + \{Ae(t) - L_o(t)He(t) + \phi(.)\}^TY(t)e(t)$$

$$\dot{V}(t) = -e^TY(t)[AP + PA^T + GQG^T - P_c - AP_{cd} - P_{cd}A^T]Y(t)e(t) + e^TY(t)\{Ae(t) - L_oHe(t) + \phi(.)\} + \{e^T(t)A^T - e^T(t)H^TL_o^T + \phi^T(.)\}Y(t)e(t) \quad (58)$$

Simplifying further and canceling certain common terms (without any approximations), one obtains

$$\dot{V}(t) = -e^T(t)YGQG^TYe(t) + e^T(t)YP_cYe(t) + e^T(t)YAP_{cd}Ye(t) + e^T(t)YP_{cd}A^TYe(t) + e^T(t)Y\{Ae(t) - L_oHe(t) + \phi(.)\} + \{e^T(t)A^T - e^T(t)H^TL_o^T + \phi^T(.)\}Ye(t) \quad (59)$$

$$\dot{V}(t) = -e^T(t)YGQG^TYe(t) + e^T(t)YP_cYe(t) + e^T(t)YAP_{cd}Ye(t) + e^T(t)YP_{cd}A^TYe(t) + e^T(t)YAe(t) - e^T(t)YL_oHe(t) + e^T(t)Y\phi(.) + e^T(t)A^TYe(t) - e^T(t)H^TL_o^TYe(t) + \phi^T(.)Ye(t) \quad (60)$$

Substituting the CTHF gain from (44) in (60), one obtains

$$\dot{V}(t) = -e^T(t)YGQG^TYe(t) + e^T(t)YP_cYe(t) + e^T(t)YAP_{cd}Ye(t) + e^T(t)YP_{cd}A^TYe(t) + e^T(t)YAe(t) - e^T(t)H^TR^{-1}He(t) + e^T(t)Y\phi(.) + e^T(t)A^TYe(t) - e^T(t)H^TR^{-1}He(t) + \phi^T(.)Ye(t) \quad (61)$$

Next, the following additional bounds are defined

a) It is assumed that

$\|R^{-1}\| \leq 1/r\,; \|H^T H\| \leq h^2$, (r and h are positive constants); $\|e(t)\|^2 \leq \varepsilon^2$ (62)

b) $\|P_c\| \leq b; \|AP_{cd}\| = \|P_{cd}A^T\| \leq c; \|A\| \leq a$ (63)

After substituting these bounds one gets the following form of (61) in terms of the norms;

$$\dot{V}(t) = -\frac{q_l}{p_u^2}\|e(t)\|^2 + \frac{b}{p_u^2}\|e(t)\|^2 + \frac{2c}{p_u^2}\|e(t)\|^2 + \frac{2a}{p_u}\|e(t)\|^2 + \frac{2\rho\varepsilon}{p_u}\|e(t)\|^2 - \frac{2h^2}{r}\|e(t)\|^2 \quad (64)$$

In (64), q_l is the smallest (positive) eigenvalue of the matrix GQG^T, that is positive definite and symmetric. Combining certain terms in (64), one gets

$$\dot{V}(t) = -\left\{\frac{q_l}{p_u^2} - \frac{b}{p_u^2} - \frac{2c}{p_u^2}\right\}\|e(t)\|^2 - \left\{\frac{2h^2}{r} - \frac{2a}{p_u} - \frac{2\rho\varepsilon}{p_u}\right\}\|e(t)\|^2 \quad (65)$$

$$\dot{V}(t) = -\left\{\left[\frac{q_l}{p_u^2} - \frac{b}{p_u^2} - \frac{2c}{p_u^2}\right]; \left[\frac{2h^2}{r} - \frac{2a}{p_u} - \frac{2\rho\varepsilon}{p_u}\right]\right\}\|e(t)\|^2 \quad (66)$$

For $\|e(t)\| \leq \varepsilon = k$, the following condition from (66) results

$$\dot{V}(t) = -\left\{q_l > (b + 2c); k < \frac{h^2 p_u - ar}{r\rho}\right\}\|e(t)\|^2 \quad (67)$$

Since, various constants and bounds (all defined earlier) appearing in the {.;.} of (67) are positive, then for the specified conditions on q_l and k in (67), it is seen that the time derivative of the Lyapunov energy functional is locally negative definite as in (67). Hence, the error dynamics of the nonlinear observer with the gain and state error 'covariance' Gramian from the continuous time HI filter, are locally asymptotically stable; and interestingly this result also establishes that the continuous time HI filtering (CTHF) algorithm would be asymptotically stable. The latter is true because the asymptotic convergence result is based on the observer error dynamics that use the CTHF gain and the Gramian matrix from the CTHF. This is a novel interpretation in this paper.

3.6 Performance evaluation of the CTHF

The CTHF algorithm is implemented in MATLAB and validated using simulated data. The state space model considered is given as

$$\dot{x}(t) = Ax(t) + Bu + w \quad (68)$$

The system matrices are

$$A = \begin{bmatrix} 0.06 & -2.0 \\ 0.8 & -0.8 \end{bmatrix}; B = \begin{bmatrix} -0.6 \\ 1.5 \end{bmatrix} \quad (69)$$

The measurement model is given as

$$z(t) = Hx(t) + v \quad (70)$$

The simulated data for 10 sec. are generated using the models, (69), (70), with a sampling interval of 0.05 sec. Appropriate additive process noise in states and measurement noise are used. Control input signal u(.) is a double that excites the modes of the system (69). The initial conditions for the states and the Gramians are appropriately chosen. The % fit errors (metrics) of the measurement residuals and the states are computed as PF/SE=100*cov(of measurements or state errors)/cov(of true or actual means. or state). If covariance measure is found to be ill-conditioned, then 'norm' can be used instead. Also, the H-infinity norm is evaluated as

$$HI(norm) = \frac{\sum_{K=0}^{N}\left(\hat{x}(k) - x(k)^T\right)\left(\hat{x}(k) - x(k)\right)}{\left(\hat{x}_0 - x^T\right)P_0\left(\hat{x}_0 - x\right) + \sum_{k=0}^{N} w^T(k)w(k) + \sum_{k=0}^{N} v^T(k)v(k)} \quad (71)$$

The HI norm is the ratio of the output error (state estimate error) energy to the total input energies of the disturbances; this includes the error in the state initial condition and all the noise variances. The dynamic equations for simulation as well as filtering algorithm are solved by RK4 method of integration. The RTD equation (45) is solved by using the transition matrix method (Gelb, 1974). The performance metrics are given in Table 1; from where it can be seen that the CTHF algorithm performs very satisfactorily. Figure 1-3 show the CTHF performance; which shows very satisfactory trajectory matching and the state errors are within their theoretical bounds. Figure 3 depicts the time history of the norm of P(t) which corroborates the asymptotic convergence of the observer as well as the continuous time H-Infinity filtering algorithm. Extensions of the presented analytical results to the applications in the domain of multi sensory data fusion can be easily pursued (Vershinin, 2002; Raol, 2015).

Table 1
Percentage performance metrics for CTHF algorithm

HI norm	**PFE residuals**
0.0269	4.8968
PSE state x1	**PSE state x2**
0.2038	0.1213

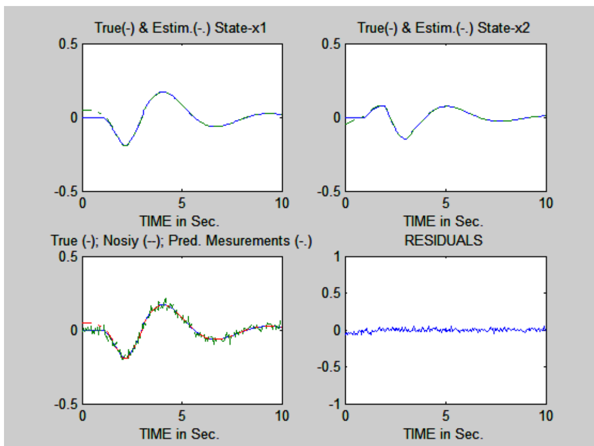

Figure 1: Time histories match (CTHF)/Residuals

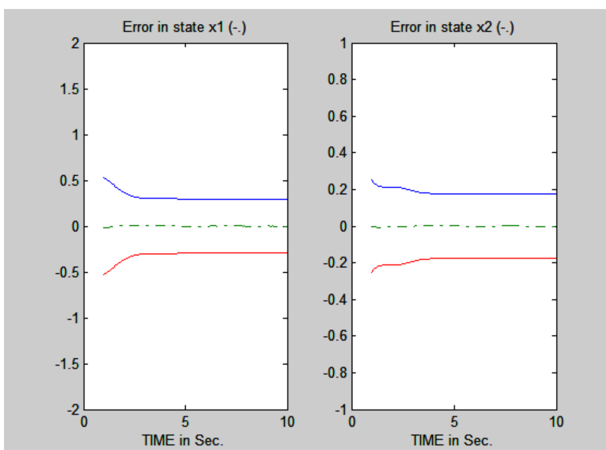

Figure 2: State errors with their bounds (CTHF)

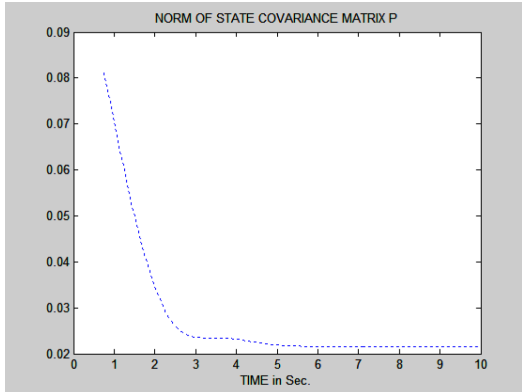

Figure 3: Norm of P(.) showing the convergence

4. Conclusion

H-infinity a posteriori filter (HIPF) has been converted to a continuous time H-infinity (HI) filter. Then, an observer is presented which uses the gain and the state error Gramian from the continuous time HI filter (CTHF). Asymptotic stability result of the observer's error dynamics is derived using the Lyapunov energy functional. The performance of the CTHF is evaluated using numerical simulations carried out in MATLAB. These results establish the local stability of the underlying CTHF. This type of result is novel in the literature on HI theory of filters and the observers. Such studies have utilization in applications of the CTHF algorithm and observers in communications systems, wireless sensor networks, mechanical/aerospace engineering (aircraft trajectory & parameter estimation and target tracking), and robotics.

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Statement of authorship
The author(s) have a responsibility for the conception and design of the study. The author(s) have approved the final article.

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Biography of Authors

	Swetha Amit, born in Bangalore on 28th October 1983, Completed her B.E in Electronics and Communication, AIT, Chikmagalur in 2005, M.Tech in Communication Systems, RVCE, Bangalore in 2009. Currently Pursuing Ph.D. in Electronics Engineering from Jain University. Presently working as Assistant Professor in Telecommunication Engineering department, Ramaiah Institute of Technology since August 2012. Published paper in Control Engineering and Antenna Design. Secured Gold medal in M.Tech. *Email: swetha.amit@gmail.com*
	Jitendra, R, Raol has BE and ME degrees from M. S. University of Baroda (1971/1974) and Ph.D. from McMaster University, Canada, 1986. He worked in CSIR-NAL, Bangalore from 1975 to 1981 on human pilot modeling in fix- and motion-based simulators, and from 1986 to 2007, on parameter estimation, filtering, and data fusion. He retired as Scientist-G & Head, FMCD (CSIR-NAL) in July 2007. He has been a senior member of the IEEE (USA), the fellow of IEE/IET (UK), and he is life fellow of Aero. Soc. of India, and a life member of Syst. Soc. of India. He has won several awards/prizes. He has severed as a chairman and member of several technical/administrative committees and has also evaluated several doctoral theses. He is a reviewer for several national/international technical and research journals. He has authored (singly and jointly) six books, published by IEE, UK, and CRC Press, USA. He has published more than 130 research papers. He has carried out several sponsored R&D projects in several areas. He had visited several countries on deputation to conduct R&D and to present papers at some technical conferences. His current activities include parameter estimation, nonlinear filtering, sensor data fusion, and soft computing. He has been Professor Emeritus in the depths. of Instrumentation Technology and E & C Engg., of M. S. Ramaiah Institute of Technology, Bangalore for a number of years. He is the chief editor of Control and Data Fusion e-Journal, an online open access journal started very recently. He is also an author of some literary works. *Email: raoljr@gmail.com*